MAT 578 **HOMEWORK 4** **Due: 9/20/07**

Solve three of the following problems.

15. Let X be a Banach space, let M, N be closed subspaces, and assume that N is finite dimensional.
    (a) Prove that $M + N$ is closed.
    (b) Let $\pi : X \to X/N$ be the quotient map. Prove that $\pi(M)$ is closed.
    (c) Let X and Y be Banach spaces, and let $T \in B(X, Y)$ be Fredholm. Let X_0 be a closed subspace of X. Prove that TX_0 is closed.

16. For a bounded operator T on a Banach space X, define the *Weyl spectrum* of T to be
$$\sigma_W(T) = \cap\big\{\sigma(T + E) : E \in K(X)\big\}.$$
    (a) Prove that $\sigma_W(T) = \emptyset$ if X is finite dimensional.
    (b) Prove that $\sigma_e(T) \subseteq \sigma_W(T)$ if X is infinite dimensional.

17. Let S be the unilateral rightward shift on ℓ^2. Prove that $\sigma_e(S) = \mathbb{T}$ and that $\sigma_W(T) = \overline{\mathbb{D}}$ (where $\mathbb{D}$ is the open unit disc in $\mathbb{C}$).

18. Let X and Y be Banach spaces.
    (a) Let $T \in F(X, Y)$. Prove that T is a compact perturbation of an invertible operator if and only if $\operatorname{ind}(T) = 0$.
    (b) Let S, $T \in F(X, Y)$. Prove that $\operatorname{ind}(S) = \operatorname{ind}(T)$ if and only if there is an invertible operator $U \in B(Y)$ such that $S - TU \in K(X, Y)$.