MAT 578 **HOMEWORK 3** **Due: 9/13/07**

Solve three of the following problems.

10. Let T be the operator on $\mathbb{C}^3$ whose matrix relative to the standard basis is
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$
    (a) Find the spectral projection of T corresponding to the subset $\{0\}$ of $\sigma(T)$.
    (b) Let $f(z) = 1 - z$. Then $f\big|_{\sigma(T)} = \chi_{\{0\}}$, but $f(T)$ is not the answer to part (a). Why?

11. Let $z_0 \in \mathbb{C}$ and $R > 0$, and let $D = \big\{z \in \mathbb{C} : |z - z_0| < R\big\}$. Let A be a unital Banach algebra, let $a \in A$, and assume that $\sigma(a) \subseteq D$. Let $f : D \to \mathbb{C}$ be analytic, so that f has a power series expansion $f(z) = \sum_{n\geq 0} c_n(z - z_0)^n$ converging absolutely, and uniformly on compact subsets, in D. Prove that $f(a) = \sum_{n\geq 0} c_n(a - z_0)^n$.

12. (a) Let A be a complex algebra and let I be a proper ideal of A. Prove that I is a maximal ideal if and only if the quotient algebra A/I is simple.
    (b) Prove that the complex algebra $M_n(\mathbb{C})$ is simple.
    (c) Let A be a unital Banach algebra, let $n \in \mathbb{N}$, and let $\omega : A \to M_n(\mathbb{C})$ be a surjective homomorphism of complex algebras. Prove that ω is continuous.

14. Let X and Y be Banach spaces, and let $T \in B(X, Y)$. Suppose that TX has finite codimension. Prove that TX is closed.