MAT 578 **HOMEWORK 10** **Due: 11/1/07**

Solve three problems from among the following, and from among any unworked problems on assignment 9.

42. Let $T \in B(H)$ be a normal operator. Let $\xi \in H$ and $\lambda \in \mathbb{C}$ such that $T\xi = \lambda\xi$. Prove that for any bounded Borel function $f \in B\big(\sigma(T)\big)$ we have $f(T)\xi = f(\lambda)\xi$.
43. Let P, $Q \in B(H)$ be projections. Prove that $P \leq Q$ if and only if $PH \subseteq QH$.
44. Let $\{q_i : i \in I\}$ be a family of projections in $B(H)$, such that $q_i q_j = 0$ if $i \neq j$. Prove that $\sum_{i \in I} q_i$ converges in the strong operator topology, with limit equal to the projection onto the closed linear span of the ranges of the $\{q_i\}$.
45. Let $0 \leq t \leq 1$. Prove that there is a sequence of projections on ℓ^2 that converge to $t1$ in the weak operator topology.