MAT 472 — Spielberg **HOMEWORK 9** **Due: 11/2/06**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution.

33. Let $f : \mathbf{R} \to \mathbf{R}$ be twice differentiable, suppose that $f'' - f = 0$, and suppose that $f(0) = f'(0) = 0$. Use Taylor's theorem to prove that $f = 0$.

34. Let $E = \{1/n : n \in \mathbf{N}\}$. Define $f : [0, 1] \to \mathbf{R}$ by $f = \chi_E$ (the characteristic function of E). Prove that f is Riemann integrable.

35. Let f be Riemann integrable on $[a, b]$. Prove that f^2 is Riemann integrable on $[a, b]$. (Hints: if $|f|$ is bounded by K on $[a, b]$, and $[x_{i-1}, x_i]$ is a subinterval on which f has maximum M_i and minimum m_i, then for any s, $t \in [x_{i-1}, x_i]$, show that $f(s)^2 - f(t)^2 \leq 2K(M_i - m_i)$. Use the lemma from lecture:

> **Lemma.** Let $f : [c, d] \to \mathbf{R}$ be bounded, and let m, M be the infimum, respectively supremum, of f on $[c, d]$. Then $M - m = \sup\{f(s) - f(t) : s, t \in [c, d]\}$.)

36. Let $f : [a, b] \to \mathbf{R}$ be continuous, and assume that $f \geq 0$ on $[a, b]$. Assume further that there exists $x_0 \in [a, b]$ such that $f(x_0) > 0$. Prove that $\int_a^b f(x)\,dx > 0$.