MAT 472 — Spielberg **HOMEWORK 6** **Due: 10/5/06**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution.

21. Suppose that A is a connected subset of a metric space, and let $A \subseteq B \subseteq \overline{A}$. Prove that B is connected.

22. Let A and B be connected subsets of a metric space.

(i) Suppose that $A \cap B \neq \emptyset$. Prove that $A \cup B$ is connected.

(ii) Is it necessarily the case that $A \cap B$ is connected? Prove your answer.

23. Let $\{E_i\}_{i \in I}$ be a family of connected subsets of a metric space. Suppose that for all j, $k \in I$ there exist $i_1, \ldots, i_n \in I$ such that

$$i_1 = j, \quad i_n = k, \quad \text{and } E_{i_\alpha} \cap E_{i_{\alpha+1}} \neq \emptyset, \text{ for } 1 \leq \alpha < n.$$

Prove that $\bigcup_{i \in I} E_i$ is connected. (Hint: If A is a clopen subset of the union, consider the set $J = \{i \in I : A \cap E_i \neq \emptyset\}$.)

24. Let (X, d) be a connected metric space, and suppose that X has more than one element. Prove that X is uncountable. (Hint: suppose that $a \neq b$ in X. For $0 < t < d(a, b)$ let $A_t = \{x \in X : d(a, x) = t\}$. Prove that $A_t \neq \emptyset$.)