MAT 472 — Spielberg **HOMEWORK 3** **Due: 9/14/06**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution.

9. Let E be a subset of a metric space.
(i) Prove that E' is a closed set.
(ii) Prove that $E' = (\overline{E})'$.
(iii) Is it necessarily the case that $E' = (E')'$? Prove your answer.

10. Let (X, d) be a metric space. For $E \subseteq X$ we define the *boundary* of E by

$$\partial E = \overline{E} \cap \overline{X \setminus E}.$$

Prove the following:
(i) X is the disjoint union of $\operatorname{int}(E)$, $\operatorname{int}(X \setminus E)$, and ∂E.
(ii) E is closed if and only if $\partial E \subseteq E$.
(iii) E is open if and only if $\partial E \cap E = \emptyset$.

11. Let (X, d) be a metric space, and let $\{E_i\}_{i \in I}$ be a family of subsets of X.
(i) Prove that $\bigcup_{i \in I} \overline{E_i} \subseteq \overline{\bigcup_{i \in I} E_i}$.
(ii) Prove that equality holds in (i) if the index set I is finite.
(iii) Prove that equality need not hold in (i) if the index set is infinite.

12. (i) Prove that $\overline{B_r(a)} = \overline{B}_r(a)$ in $\mathbf{R}^n$.
(ii) Does the same result hold in the space X of problem 6? Prove your answer.