MAT 472 **HOMEWORK 1** **Due: 8/31/06**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution.

1. Prove the following. Let $x \in \mathbf{R}$. Suppose that there exist positive real numbers p and q such that for every real number ϵ with $0 < \epsilon < p$ we have $|x| < \epsilon q$. Then $x = 0$.

2. For A and B subsets of $\mathbf{R}$ we define

$$A + B = \{x + y : x \in A \text{ and } y \in B\}.$$

Let A and B be nonempty subsets of $\mathbf{R}$ that are bounded below. Prove that

$$\inf(A + B) = \inf(A) + \inf(B).$$

3. Let X and Y be non-empty sets, and let $f : X \times Y \to \mathbf{R}$ be a bounded function.

(i) Prove that

$$\sup_{y \in Y} \left(\inf_{x \in X} f(x, y) \right) \leq \inf_{x \in X} \left(\sup_{y \in Y} f(x, y) \right).$$

(ii) Give an example (with proof) where the inequality is strict.

4. Let $a_1, a_2, \ldots \in \mathbf{R}$. Suppose that the set

$$\left\{ n \ : \ \frac{|a_n|}{n} \geq 1 \right\}$$

is infinite. Prove that the set

$$\left\{ m \ : \ \frac{|a_1 + a_2 + \cdots + a_m|}{m} \geq \frac{1}{2} \right\}$$

is also infinite.