1. Let A, B, and C be sets. Prove that $(A \setminus B) \cup C = (A \cup C) \setminus (B \setminus C)$.

2. Let A and B be sets. Prove that $(A \times B)' = (A' \times B) \cup (A \times B') \cup (A' \times B')$.

3. Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \dfrac{1}{3}n(n+1)(n+2)$.

4. Prove that $2^n < n!$ for $n \geq 4$.

5. Let p be a prime number, let a be an integer, and suppose that $a^2 \equiv_p 1$. Prove that $a \equiv_p 1$ or $a \equiv_p -1$.

6. Is the following a function from $[-1, 1]$ to $\mathbf{R}$? Prove your answer.

$$\{(\cos x, \sin x) : x \in \mathbf{R}\}.$$

7. Let M be the following relation on $\mathbf{R}$: $M = \{(x, y) \in \mathbf{R} \times \mathbf{R} \ : \ |x - y| < 1\}$. Is M reflexive? symmetric? antisymmetric? transitive? Prove your answers.

8. Let $f : \mathbf{R} \setminus \{1\} \to \mathbf{R} \setminus \{0\}$ be defined by

$$f(x) = \frac{1}{x - 1}.$$

Prove that f is bijective.

9. Suppose that $f : A \to B$ and $g : B \to A$ are such that $f \circ g = i_B$.
(i) Prove that g is one-to-one and that f is onto.
(ii) Suppose that g is onto. Prove that f is one-to-one.

10. Let $f : A \to B$ be one-to-one. Let $S \subseteq A$. Prove that $f(S') \subseteq f(S)'$.

11. Let $f : A \to B$ and let $T \subseteq B$. Prove that $f^{-1}(T') = f^{-1}(T)'$.

12. Let $f : A \to B$ and $g : B \to C$. Rewrite each of the following statements using the symbols $\forall$, $\exists$, $\to$, $\wedge$, $\vee$, $\in$, $\notin$, $=$, $\neq$. Do not use the symbols $\subseteq$, $\cap$, $\cup$, g^{-1}, f^{-1}, $f(S)$.
(i) $f(S) \subseteq g^{-1}(T)$
(ii) $g^{-1}(T) \subseteq f(S)$.

13. Prove that $\sqrt{3}$ is irrational.

14. Give the formula for $P(n, m)$. What does this number represent?

15. Give the formula for $C(n, m)$. What does this number represent?

16. State the triangle inequality for real numbers x and y. Use the triangle inequality to prove that for all real numbers x and y, we have $|x + y| \geq |x| - |y|$.

17. Prove that for any real number a, $a \cdot 0 = 0$. Use only the axioms for the real numbers, and justify each step of your proof with the axiom number.

18. Complete the definitions of the following terms.
(i) The set A is *finite* if ...
(ii) The set A is *infinite* if ...
(iii) The set A is *countably infinite* if ...
(iv) The set A is *countable* if ...
(v) The set A is *uncountable* if ...

19. Prove that $[0, 2) \sim (0, 1]$.

20. Let x be a non-zero rational number and let y be an irrational number. Prove that $(x - y)/(x + y)$ is an irrational number.