Mat 300 — Spielberg | Review for Test 2 | Fall 2005

1. Define the following:

(i) $A \subseteq B$
(ii) $A \cup B$
(iii) $A \cap B$
(iv) $A - B$
(v) $A \times B$
(vi) $\mathcal{P}(A)$
(vii) $\bigcup_{i\in I} A_i$
(viii) $\bigcap_{i\in I} A_i$
(ix) $A \cap B = \emptyset$
(x) $f : A \to B$
(xi) $f_*(X)$
(xii) $f^*(Y)$
(xiii) injective function
(xiv) surjective function
(xv) bijective function
(xvi) left inverse of a function
(xvii) right inverse of a function
(xviii) inverse of a function

2. Prove the following statements (about sets):

(i) $\{n^2 - n - 1 \mid n \in \mathbf{Z}\} \subseteq \{2k - 1 \mid k \in \mathbf{Z}\}$.
(ii) $A - (B - C) = (A - B) \cup (A \cap C)$
(iii) $(A \times B) - (C \times D) = \big[A \times (B - D)\big] \cup \big[(A - C) \times (B \cap D)\big]$
(iv) $B \cup \big(\bigcap_{i\in I} A_i\big) = \bigcap_{i\in I}(B \cup A_i)$

3. Let $f : A \to B$ be a function, and let $X \subseteq A$, $Y \subseteq B$.

(i) Prove that $f^*(B - Y) = A - f^*(Y)$.
(ii) Prove that $f_*(A - X) \supseteq f_*(A) - f_*(X)$.
(iii) Give counterexamples to show that neither $f_*(A - X) \supseteq B - f_*(X)$ or $f_*(A - X) \subseteq B - f_*(X)$ need hold in general.

4. Let $h : [0,\infty) \to [0,\infty)$ be given by $h(x) = \dfrac{x}{x+1}$.

(i) Find a left inverse for h (and prove that it is a left inverse).
(ii) Prove that the left inverse for h that you found in part (i) is not a right inverse.

5. Let $f : \mathbf{R} - \{0\} \longrightarrow \mathbf{R} - \{2\}$ be given by $f(x) = \dfrac{1}{x} + 2$. Prove that f is bijective.

6. Let $f : A \to B$. Suppose that f has a right inverse. Prove that f is surjective.

7. Let $g : A \to B$. Suppose that g is injective. Let Z be a set, and let $p : Z \to A$ and $q : Z \to A$ be functions such that $g \circ p = g \circ q$. Prove that $p = q$.