In problems 1–32, prove that there is a bijective function from the first set to the second. If you give the function explicitly, prove that it is one-to-one and onto. We use the following abbreviations: $\mathbf{E} = \{2, 4, 6, 8, \ldots\}$ is the set of positive even integers, $\mathbf{F} = \{1, 3, 5, 7, \ldots\}$ is the set of positive odd integers, and as usual $\mathbf{N} = \{1, 2, 3, 4, \ldots\}$ is the set of natural numbers, $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of integers, $\mathbf{Q}$ is the set of rational numbers, and $\mathbf{R}$ is the set of real numbers. We also use the notation $[a, b]$, (a, b), $[a, b)$, etc., for intervals in $\mathbf{R}$.

In working problems 1–30, you may use the existence of the bijections asserted in any problems with lower number.

1. $\mathbf{N}$ and $\mathbf{E}$.

2. $\mathbf{N}$ and $\mathbf{F}$.

3. $\mathbf{E}$ and $\mathbf{F}$.

4. $\mathbf{N}$ and $\mathbf{N} \cup \{0\}$.

5. $\mathbf{N}$ and $\mathbf{Z}$.

6. $[1, 2]$ and $[2, 3]$.

7. $[0, 1]$ and $[0, 2]$.

8. $[1, 2]$ and $[3, 5]$.

9. $[2, 5]$ and $[3, 10]$.

10. $[-1, 1]$ and $[-1, 2]$.

11. $[0, 1]$ and $[a, b]$ (assume that $a < b$).

12. $[a, b]$ and $[c, d]$ (assume that $a < b$ and $c < d$).

13. $[0, 1)$ and $(0, 1]$.

14. $[-1, 1)$ and $(-1, 2]$.

15. $[0, 2)$ and $[5, 6) \cup [8, 9)$.

16. $[0, 1)$ and $[0, 2) \cup [4, 8)$.

17. $(0, 1)$ and $(1, 2) \cup [3, 5) \cup [6, 9)$.

18. $(0, \infty)$ and $\mathbf{R}$.

19. $(0, 1]$ and $[1, \infty)$.

20. $[0, \infty)$ and $[1, \infty)$.

21. $[0, 1)$ and $[0, \infty)$.

22. $(0, 1)$ and $(0, \infty)$.

23. $(0, 1)$ and $\mathbf{R}$.

24. $[0, \infty)$ and $[0, 1) \times \mathbf{N}$.

25. $\mathbf{R}$ and $[0, 1) \times \mathbf{Z}$.

26. $[0, \infty)$ and $\mathbf{R}$.

27. $[0, 1)$ and $(0, 1)$.

28. $[0, 1]$ and $(0, 1)$.

29. $[0, 1] \times [0, 1]$ and $(0, 1) \times (0, 1)$ (the unit square in the plane, and the same square without its boundary).

30. $\mathbf{N}$ and $\mathbf{Q}$. (Use the fact proved in class that $\mathbf{N} \sim \mathbf{Q}_+$, where $\mathbf{Q}_+$ is the set of positive rational numbers.)

31. $\mathbf{N} \times \{0, 1\}$ and $\mathbf{N}$.

32. $\mathbf{N} \times \mathbf{N}$ and $\mathbf{N}$.

33. Prove that if $B \sim C$ then $(A \times B) \sim (A \times C)$.

34. Prove that if $A \cap B = \emptyset$, $C \cap D = \emptyset$, $A \sim C$, and $B \sim D$, then $(A \cup B) \sim (C \cup D)$.

35. Suppose that $A \sim \mathbf{N}$ and that $x \notin A$. Prove that $(A \cup \{x\}) \sim \mathbf{N}$.

36. The unit square and the unit disk in the plane are equivalent. (The unit square is the set $[0, 1] \times [0, 1]$, and the unit disk is the set $\{(x, y) : x^2 + y^2 \leq 1\}$.)