MAT 472 **HOMEWORK 5** **Due: 9/30/04**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution. If you submit some nonsense in the course of a proof, the whole thing might strike me as not worth reading.

21. Let (X, d) be a metric space. Prove that X is sequentially compact if and only if every infinite subset of X has a cluster point.

22. Let C be the Cantor set, as constructed in class.
(i) Prove that the interior of C is empty.
(ii) Prove that $C = C'$.

23. Let (X, d) be a complete metric space. For subsets A, $B \subseteq X$ define the *distance* between A and B by

$$\operatorname{dist}(A, B) = \inf\{d(x, y) \mid x \in A,\ y \in B\}.$$

(i) Prove that if A is compact, B is closed, and $A \cap B = \emptyset$, then $\operatorname{dist}(A, B) > 0$.
(ii) Prove that the conclusion of part (i) may fail if A and B are closed but neither is compact.

24. Let (X, d) be a compact metric space, and let $\mathcal{U}$ be an open cover of X. Prove that there exists a postive number ϵ such that every closed ball in X of radius ϵ is contained in a set from $\mathcal{U}$.