MAT 472 **HOMEWORK 1** **Due: 8/31/04**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution. If you submit some nonsense in the course of a proof, the whole thing might strike me as not worth reading.

Inequalities derived from the binomial theorem may be helpful for several of these problems. Do not use facts about convergence of sequences in this homework assignment.

1. Let $a_1, a_2, \ldots \in \mathbf{R}$. Suppose that the set

$$\left\{ n \ : \ \frac{|a_n|}{n} \geq 1 \right\}$$

is infinite. Prove that the set

$$\left\{ m \ : \ \frac{|a_1 + a_2 + \cdots + a_m|}{m} \geq \frac{1}{2} \right\}$$

is also infinite.

2. For A and B subsets of $\mathbf{R}$ we define

$$A + B = \{x + y : x \in A \text{ and } y \in B\}.$$

Let A and B be nonempty subsets of $\mathbf{R}$ that are bounded below. Prove that

$$\inf(A + B) = \inf(A) + \inf(B).$$

3. Let $0 < a < 1$ and let $E = \{1,\ a,\ a^2,\ a^3,\ \ldots\}$. Prove that $\inf(E) = 0$.

4. Let E be the subset of $\mathbf{R}$ given by

$$E = \left\{ \sqrt{6},\ \sqrt{6 + \sqrt{6}},\ \sqrt{6 + \sqrt{6 + \sqrt{6}}},\ \ldots \right\}.$$

(i) Prove that E is bounded.

(ii) Find $\inf(E)$ and $\sup(E)$, and prove your answers.

5. Assume that nth roots of positive real numbers exist.

(i) Let $x > 0$. Prove that

$$(1 + x)^n > \frac{1}{2}(n^2 - n)x^2, \quad n \in \mathbf{Z}^+.$$

(ii) Prove that

$$\sqrt[n]{n} - 1 < \sqrt{\frac{2}{n - 1}}$$

for $n > 1$.

(iii) Let $E = \{ \sqrt[n]{n} \ : \ n \geq 2\}$. Find (with proof) $\inf E$ and $\sup E$ (no calculators, or calculus, allowed).