MAT 572 **Homework 5** **Due Wednesday, 10/11**

Always prove your answers.

1. Let f be entire. Suppose that for some $n \geq 0$ and $R > 0$, we have $\left|f(z)\right| \leq |z|^n$ for all $|z| \geq R$. Prove that f is a polynomial.

2. Compute the following:

(i) $$\int_{|z|=r} \frac{z^2+1}{z(z^2+4)}\, dz, \quad r > 0, \quad r \neq 2.$$

(ii) $$\int_{|z-1|=1/2} \frac{z^{1/m}}{(z-1)^m}\, dz, \quad m \geq 0.$$

(iii) $$\int_{|z-1|=1/2} \frac{\log z}{z^n}\, dz, \quad n \geq 0.$$

3. Let $a \in \mathbf{C}$ and let $R > 0$ with $R \neq |a|$. Compute

$$\int_{|z|=R} \frac{1}{|z-a|^2}\, |dz|.$$

(Hint: use $z\overline{z} = R^2$ to show that $|dz| = dz/(iz)$.)

4. Let G be a region. Prove that every cycle in G is homologous to zero if and only if for every $a \in \mathbf{C} \setminus G$ there is a continuous branch of $\log(z-a)$ defined on G.