MAT 272 **Test 3** Name______________________

Each problem is worth 13 points. Explain, justify, and/or illustrate your method of solution. If Maple is used to find derivatives, state this and write down the results.

1. Find and classify the extrema of $f(x,y) = 2x^3 - 3x^2 - 12x + 2y^3 - 54y$

$f_x = 6x^2 - 6x - 12$

$6(x^2 - x - 2) = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

$f_y = 6y^2 - 54$

$6(y^2 - 9) = 0$

$y = \pm 3$

$f_{xx} = 12x - 6 \qquad f_{yy} = 12y \qquad f_{xy} = 0$

$D = (12x - 6)12y - 0$

$(-1, 3)$: $D < 0$, S.P.

$(-1, -3)$: $D > 0$, $f_{xx} < 0$, REL. MAX.

$(2, 3)$: $D > 0$, $f_{xx} > 0$, REL. MIN.

$(2, -3)$: $D < 0$, S.P.

2. Find and classify the extreme points of $f(x,y) = x^4 - xy + y^4$

$f_x = 4x^3 - y$ $\qquad$ $f_y = 4y^3 - x$

$y = 4x^3$ $\qquad$ $x = 4y^3$

$y = 4(4y^3)^3$

$y = 256y^9$

$y(256y^8 - 1) = 0$

$y = 0,\ y = \pm\frac{1}{2}$

$x = 0,\ x = \frac{1}{2},\ -\frac{1}{2}$

$f_{xx} = 12x^2$ $\qquad$ $f_{yy} = 12y^2$ $\qquad$ $f_{xy} = -1$

$D = 12x^2(12y^2) - (-1)^2$

$(0,0)$: $D < 0$, S.P.

$(\frac{1}{2}, \frac{1}{2})$: $D > 0$, $f_{xx} > 0$, REL. MIN.

$(-\frac{1}{2}, -\frac{1}{2})$: $D > 0$, $f_{xx} > 0$, REL. MIN.

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3. Find and classify the critical points of $f(x,y) = x^2 + 2y^2 - x^2 y$

$f_x = 2x - 2xy$ $\qquad$ $f_y = 4y - x^2$

$2x(1-y) = 0$ $\qquad$ $4y = x^2$

$x = 0, \quad y = 1$

$y = 0 \qquad x = \pm 2$

$f_{xx} = 2 - 2y$ $\qquad$ $f_{yy} = 4$ $\qquad$ $f_{xy} = -2x$

$D = (2-2y)4 - (-2x)^2$

$(0,0)$: $D > 0$, $f_{xx} > 0$, REL. MIN.

$(2,1)$: $D < 0$, S.P.

$(-2,1)$: $D < 0$, S.P.

4. A dietician wished to determine if a linear relationship exist between the height of a female and her weight. The table below gives the heights and weights of 9 females aged 18–24. The heights are measured in centimeters and the weights are measured in kilograms. Find a linear regression equation which could be used to predict the weight of a female given her height and use it to predict the weight of a woman whose height is 185 cm.

Height	152	157	162	165	170	173	175	178	180
Weight	47	50	52	54	56	58	61	63	65

Formulas (if needed): $b = \dfrac{\sum x^2 \sum y - \left(\sum x\right)\left(\sum xy\right)}{n\sum x^2 - \left(\sum x\right)^2}$ $\qquad m = \dfrac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$

$y = -48.33 + .622x$

USE MAPLE OR A CALCULATOR

$y = -48.33 + .622(185)$

66.74 kg

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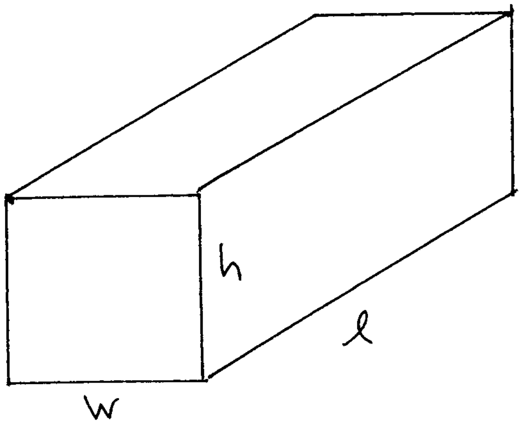
5. Suppose a post office will not mail a rectangular box if the sum of its length and girth (the perimeter of a cross section that is perpendicular to the length) is more than 108 inches. Find the dimensions of the box of maximum volume that can be mailed.



$$V = lwh$$

$$108 = l + 2w + 2h$$

$$\nabla V = \lambda \nabla G$$

$$(wh, lh, lw) = \lambda (1, 2, 2)$$

$$wh = \lambda \qquad lh = 2\lambda \qquad lw = 2\lambda$$

$$wh = \lambda \qquad \frac{lh}{2} = \lambda \qquad \frac{lw}{2} = \lambda$$

$$\frac{lh}{2} = \frac{lw}{2}$$

$$h = w$$

$$wh = \frac{lh}{2}$$

$$l = 2w$$

$$2w + 2w + 2w = 108$$

$$6w = 108$$

$$w = 18$$

$$h = 18$$

$$l = 36$$

$$V(18, 18, 36) = 11664$$

$$V(26, 26, 4) = 2704$$

$\therefore$ MAX.

6. Optimize $f(x, y) = xy + yz + xz$ subject to the constraint $x + y + z = 1$.

$$\nabla f = \lambda \nabla g$$

$$(y+z,\ x+z,\ x+y) = \lambda(1,1,1)$$

$$y+z = \lambda \qquad x+z = \lambda \qquad x+y = \lambda$$

$$y+z = x+z \qquad y+z = x+y$$

$$y = z \qquad z = x$$

$$x = y = z$$

$$f\left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right) = \tfrac{1}{3}$$

$$f\left(\tfrac{1}{2}, \tfrac{1}{2}, 0\right) = \tfrac{1}{4} \qquad \therefore \text{ MAXIMUM}$$