

MAT451 Midterm Exam

Name:

Student ID:

Show All Work Necessary to Justify Your Answer! No Work, No Credit!

1. A two-player normal form game has payoffs given in the following table.

$$\begin{bmatrix} * & A & B & C & D \\ R & -1,2 & 1,2 & 2,3 & 2,4 \\ S & 0,3 & 2,1 & 1,1 & 4,0 \\ T & -1,2 & 2,2 & 2,3 & 1,3 \end{bmatrix}.$$

a. Find all dominance relations among the strategies and indicate whether they are strict or weak.

A and C weakly dominate B.

b. Find player 1's best response set; find player 2's best response set; find all pure strategy Nash equilibria.

$$\begin{aligned} A_1 &= \{(S, A), (S, B), (T, B), (R, C), (T, C), (S, D)\} \\ A_2 &= \{(R, D), (S, A), (T, C), (T, D)\} \\ A_1 \cap A_2 &= \{(T, C), (S, A)\} = \text{Nash} \end{aligned}$$

2. Consider (3.3&4.8) Competition on Main Street. Recall store owners locating at the same place must share equally. Assume store owners can locate stores only at $\{0, 1/4, 1/2, 3/4, 1\}$.

a. Suppose there are 2 store owners. Fill in those entries in owner-1's payoff matrix below not already occupied with an asterisk. Is locating a store at 0 a strictly dominated strategy? Explain!

$$\begin{bmatrix} * & 0 & 1/4 & 1/2 & 3/4 & 1 \\ 0 & 1/2 & 1/8 & 1/4 & 3/8 & 1/2 \\ 1/4 & 7/8 & 1/2 & 3/8 & 1/2 & 5/8 \\ 1/2 & * & * & * & * & * \\ 3/4 & * & * & * & * & * \\ 1 & * & * & * & * & * \end{bmatrix}.$$

Entries in 1/4th row are strictly larger than corresponding terms in 0th row so locating at 1/4 strictly dominates locating at 0!

b. Suppose there are 3 store owners. Is $(1/2, 1/2, 1/2)$ Nash? Explain!

$\pi_1(1/4, 1/2, 1/2) = 3/8 > \pi_1(1/2, 1/2, 1/2) = 1/3$ so $(1/2, 1/2, 1/2)$ is not Nash!

3. Consider Strategic Voting whose game tree is shown below (exactly as on page 397 of text). Order each player's nodes starting from the top so, for example, player 3 strategy $nynn$ means vote no at top node, vote yes on second from top node, etc. Is $(y, ny, nyyn)$ Nash? Justify your argument!

Players 2 and 3 are happy but Player 1, who gets $\pi_1(y, ny, nyyn) = b - c$ from playing y could get $\pi_1(n, ny, nyyn) = b$ by switching to n because Player 2 must do y and Player 3 must do y . So $(y, ny, nyyn)$ is not Nash.

4. Two players agree to play **two rounds** of Prisoners Dilemma using the payoff matrix below for each round. However, if **one or both** players defect (D) in the first round then the game terminates. As usual, on a given round, players choose C or D simultaneously.

$$\begin{bmatrix} * & C & D \\ C & 4,4 & 1,5 \\ D & 5,1 & 2,2 \end{bmatrix}.$$

a. Draw a game tree, labeling all nodes and branches appropriately, indicating information sets where necessary, and payoffs.

b. Describe the strategy set for each player. Don't just give letters or symbols but actually describe the strategies.

CC=cooperate on both rounds.

CD=cooperate on round one, defect on round two.

DC=defect on round one, cooperate on round two.

DD=defect on both rounds.

c. Find a subgame perfect Nash equilibrium. Find all Nash equilibrium.

There are only two subgames-the whole game and the "Second Round subgame".

Numbering strategies $1 = CC, 2 = CD, 3 = DC, 4 = DD$, we get the payoff matrix for player 1:

$$\begin{bmatrix} 8 & 5 & 1 & 1 \\ 9 & 6 & 1 & 1 \\ 5 & 5 & 2 & 2 \\ 5 & 5 & 2 & 2 \end{bmatrix}.$$

So $(CD, CD), (DD, DD), (DC, DC)$ are symmetric pure Nash. $(CD, CD), (DD, DD)$ are subgame perfect since both defect on the last round, the only Nash for one-round Prisoner's Dilemma !

5. The player-1 payoff matrix for a two-player, symmetric game with three pure strategies is given below.

$$A = \begin{bmatrix} 100 & 60 & -200 \\ 60 & 36 & -80 \\ 200 & 80 & -100 \end{bmatrix}.$$

a. Use the Fundamental Theorem to **write down all equations and inequalities** satisfied by a mixed Nash equilibrium of the form $x^* = (0, x_2, x_3)$.

$$\begin{aligned} 60x_2 - 200x_3 &\leq \lambda \\ 36x_2 - 80x_3 &= \lambda \\ 80x_2 - 100x_3 &= \lambda \\ x_2 + x_3 &= 1 \end{aligned}$$

b. Solve your equations for x_2, x_3 and the payoff. Is there such a Nash?

Solve the last 3 equations to get:

$$x_2 = 5/16, \quad x_3 = 11/16, \quad \lambda = -700/16.$$

Then check that the first inequality holds. It does! So there is such a Nash! Actually, since strategy 1 is strictly dominated, so can be eliminated, the inequality will hold!