

# MAT 420 Final Exam

Kostelich, Dec. 6, 2007

This is a take-home exam. You may use any books or notes, plus any other online resources that you wish, but you may not discuss your work with any person. You may adapt code from any freely available source, such as Netlib, **provided that you acknowledge the source**. We will assemble in the computer lab (ECA 221) during the regularly scheduled final exam period (Thursday, Dec. 6, 12:20–2:10 p.m.). You must bring with you:

- A stapled printout of your programs with your printed name at the top.
- The statement “I have not received help from any person on this examination” must be handwritten and signed by you on the first page of your printout.

During the exam period, I will ask you to demonstrate that your program compiles and gives correct answers for selected input data. There are two problems, each worth 30 points.

1. Linear interpolation is a simple way to approximate smooth functions whose values are known only on a lattice of points. For example, the specific volume  $V$  of steam is a smooth function of temperature  $T$  and pressure  $P$ . Recall that the linearization of  $V = V(T, P)$  around the point  $(T_0, P_0)$  is

$$V(T_0 + \Delta T, P_0 + \Delta P) = V(T_0, P_0) + \frac{\partial V}{\partial T} \Delta T + \frac{\partial V}{\partial P} \Delta P.$$

Bilinear interpolation refers to the case where we approximate two partial derivatives,  $\partial V/\partial T$  and  $\partial V/\partial P$ , by finite differences to estimate the value of a function  $V(T, P)$  given experimentally determined values in a table.

The following table gives experimentally measured values of the specific volume of superheated steam (in  $\text{m}^3/\text{kg}$ ) as a function of temperature in degrees Celsius and pressure in atmospheres:

	240°	260°	280°	300°	320°	340°
6	0.393	0.410	0.426	0.443	0.459	0.475
7	0.336	0.350	0.365	0.379	0.393	0.407
8	0.293	0.305	0.318	0.331	0.343	0.355
9	0.259	0.271	0.282	0.293	0.304	0.315

For example, we might approximate

$$\frac{\partial V}{\partial T}(280, 6) \approx \frac{0.443 - 0.426}{20} = 0.00085$$

and

$$\frac{\partial V}{\partial P}(280, 6) \approx \frac{0.365 - 0.426}{1} = -0.063.$$

Then we would approximate  $V(290, 6.3)$  as

$$V(290, 6.3) \approx 0.426 + (0.00085)(290 - 280) + (-0.063)(6.3 - 6) = 0.416$$

to three significant figures.

Write a Fortran or C++ program that uses bilinear interpolation to approximate the specific volume of superheated steam using the data in the table and, given a list of  $(T, P)$  pairs, returns the interpolated value of  $V(T, P)$  using bilinear interpolation. You may assume that  $6 \leq P \leq 9$  and  $240 \leq T \leq 340$ , and no other error checking is needed. In lab on Thursday, I will present you with some  $(T, P)$  pairs and will ask you to demonstrate that your program correctly interpolates the table values.

2. One of the problems in graphing numerical data is to find “nice” values with which to label the coordinate axes. For example, if the  $x$  coordinates of the data points to be plotted all lie between 0.17 and 9.93, then it would be reasonable to have the  $x$  axis go from 0 to 10 with tick marks at 0, 1, 2, ..., 10. There are several reasonable heuristics that you might use. The lower and upper limits should be chosen so that all the data fits and the graph is pleasingly full. (For instance, given values from 0.17 and 9.93, you don’t want the axis to extend from  $-10$  to  $20$  because the data would occupy only a small region in the middle.) It’s also useful to have enough tick marks to pepper the axis but not so many that all the labels run into each other. (For instance, you might want to have at least 4 labels but no more than 12.) In addition, the lower and upper bounds of the axes should be chosen so that

the tick increments are conveniently rounded values, such as various integer multiples or fractions like 0.5, 0.25, or 0.1 as appropriate. For example, if the data run from  $-0.93$  to  $0.87$ , then you might want to have the axis extend from  $-1$  to  $1$  with increments of  $0.25$  or  $0.2$ ; data from  $1750$  to  $2000$  would require no rounding and a tick increment of  $50$  is fine.

Using any of Python, Fortran, or C++, write a program that reads two values,  $x_{\min}$  and  $x_{\max}$ , from the standard input. Your program should output the values that you would display on a graph axis for data with these limits. For instance, if  $x_{\min} = 0.01$  and  $x_{\max} = 0.97$ , you might choose axes limits from  $0$  to  $1$  and output

0.0, 0.25, 0.5, 0.75, 1.0

(i.e., an increment of  $0.25$  between tick marks) or

0.0, 0.1, 0.2,  $\dots$ , 1.0

(i.e., an increment of  $0.1$  between tick marks). Your algorithm should give satisfactory results for  $10^{-6} \leq x_{\max} - x_{\min} \leq 10^6$ . In your code, briefly describe the heuristics that you have developed to choose  $y_{\min}$ ,  $y_{\max}$ , and the tick increment  $\delta$ .

In lab on Thursday, I will give you some  $x_{\min}$ ,  $x_{\max}$  pairs and ask you to run your program. Your grade will be determined in large part by how well it chooses rounded values. Please check your code with example data like the following:

- $x_{\min} < x_{\max} < 0$
- $x_{\min} < 0 < x_{\max}$
- $|x_{\min}|$  and  $|x_{\max}|$  both very small (say on the order of  $10^{-5}$ ) and very large ( $10^6$  or so)
- Values like  $x_{\min} = 2.1$  and  $x_{\max} = 3.1$ , which are not exactly representable in binary arithmetic.

Please email a copy of your codes to `mat420hw` at `gmail.com`. Include your codes as a tar file attachment. Please indicate in the body of your email the names of the files, any necessary makefiles or header files, and any other information that I would need to compile and run your programs myself.