In-class exercises, May 4, 2009

These problems will not be collected or graded. They are intended to guide your study for the final exam.
Final exam reminder: Thursday, May 6 from 7:10–9:00 p.m. in PSA 109.
Office hours: 1:30–3:00 p.m. today and 10:30–12 and 1–2 tomorrow.

1. (Problem 10, p. 606) Let $\mathbf{F} = x^2\mathbf{i} + (x^2y - 2xy)\mathbf{j} - x^2z\mathbf{k}$. Does there exist a vector field $\mathbf{G}$ such that $\mathbf{F} = \nabla \times \mathbf{G}$? (Hint: see Theorem 8 in Section 8.3.)

2. (Problem 11, p. 606) Let $\mathbf{a}$ be a constant vector and let $\mathbf{F} = \mathbf{a} \times \mathbf{r}$. Is $\mathbf{F}$ conservative? If so, find a potential function for it.

3. (Problem 13, p. 606) Let $f(x,y,z) = 3xye^{z^2}$. Compute

$$\int_C \nabla f \cdot d\mathbf{r}$$

where C is the curve parametrized by $\mathbf{r}(t) = (3\cos^3 t, \sin^2 t, e^t)$, $0 \le t \le \pi$.

4. What is the area of the triangle whose vertices are $(2,0,0)$, $(0,3,0)$, and $(0,0,6)$?

5. (Problem 19, p. 607) Evaluate

$$\int_C (x+y)\,dx + (2x-z)\,dy + (y+z)\,dz$$

where C is the perimeter of the triangle in the previous problem. (Consider a theorem that lets you find the answer without doing any integration.)