In-class exercises, April 21, 2009

1. (Problem 26, p. 516) Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $F = (x, y, -y)$ and S is the cylindrical shell defined by $x^2 + y^2 = 1$, $0 \leq z \leq 1$, with normal pointing out of the cylinder.

2. (Problem 15, p. 530) Find the area enclosed by the ellipse $x^2/a^2 + y^2/b^2 = 1$. (Hint: use the area formula derived from Green's theorem.)

3. Stokes' Theorem states that for a smooth vector field $\mathbf{F}$ and an orientable surface S whose boundary is a simple closed curve,

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}.$$

   Use Stokes' theorem to redo the following problem from last week.

   (Problem 5, p. 497) Evaluate $\displaystyle\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where S is the surface $x^2 + y^2 + 3z^2 = 1$, $z \leq 0$, and $\mathbf{F}$ is the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + zx^3y^2\mathbf{k}$. (Let $\mathbf{n}$, the unit normal, be upward pointing.)