In-class exercises, April 7, 2009

1. (Problem 10, p. 418) Find the volume determined by $z \leq 6 - x^2 - y^2$ and $z \geq \sqrt{x^2 + y^2}$.

2. (Problem 21, p. 419) Find the center of mass of the solid hemisphere determined by $x^2 + y^2 + z^2 \leq a^2$ and $z \geq 0$. (Caution: the answer in the back of the book is wrong.)

3. (Problem 4, p. 447) Let $\mathbf{r}(t)$ be a smooth path. (a) Suppose $\mathbf{F}$ is perpendicular to $\mathbf{r}'(t)$ at $\mathbf{r}(t)$. Show that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = 0.$$

   (b) If $\mathbf{F}$ is parallel to $\mathbf{r}(t)$, show that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \|\mathbf{F}\| \, ds.$$

4. (Problem 28, p. 367) Sketch the region of integration, interchange the order, and evaluate:

$$\int_1^4 \int_1^{\sqrt{x}} (x^2 + y^2) \, dy \, dx.$$