

## In-class exercises, Mar. 31, 2009

These problems are from Sections 7.1 and 7.2.

1. (Problem 3b, p. 427) Evaluate  $\int_C yz \, ds$  where  $C$  is parametrized as  $\mathbf{r}(t) = (t, 3t, 2t)$  for  $1 \leq t \leq 3$ .
2. (Problem 2d, p. 447) Evaluate  $\int_C x^2 \, dx - xy \, dy + dz$  where  $C$  is the parabola  $z = x^2$  in the  $xz$  plane from  $(-1, 0, 1)$  to  $(1, 0, 1)$ .
3. Find the integral of  $\mathbf{F} = 2xy\mathbf{i} + x^2\mathbf{j}$  around the circumference of the square with vertices  $(\pm 1, \pm 1)$ .
4. (Problem 8, p. 448) A current flowing through a wire creates a magnetic field  $\mathbf{H}$  that, in appropriate units, satisfies Ampère's Law:

$$\oint_C \mathbf{H} \cdot d\mathbf{r} = I,$$

where  $C$  is a simple closed oriented curve (such as a circle) enclosing the wire. (This is the principle behind electromagnets.)

Suppose that the wire is perpendicular to the  $xy$  plane. Experiments show that  $\mathbf{H}$  is tangent to every circle in the  $xy$  plane whose center is the axis of the wire and that  $\|\mathbf{H}\|$  is constant on every such circle  $C$ . Thus,  $\mathbf{H} = H\mathbf{T}$ , where  $\mathbf{T}$  is a unit tangent vector to  $C$  and  $H$  is some scalar. Show that  $H = I/2\pi r$ , where  $r$  is the radius of the circle  $C$  and  $I$  is the current flowing in the wire. See Figure 1.

