

In-class exercises, Mar. 24, 2009

Here's one way to evaluate the integral

$$\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_{x^2+y^2}^2 x \, dz \, dy \, dx,$$

discussed in Example 5 on p. 361. (The percent sign % is a shorthand for the most recently computed result.)

```
f := x;  the integrand
int(f, z = x^2 + y^2 .. 2);
int(%, y = 0 .. sqrt(2-x^2));
int(%, x = 0 .. sqrt(2));
```

These problems are from Section 6.2.

1. (Problem 13, p. 391) Integrate $ze^{x^2+y^2}$ over the cylinder $x^2 + y^2 \leq 4$, $2 \leq z \leq 3$.
2. (Problem 14, p. 391) Let D be the unit disk. Express $\iint_D (1 + x^2 + y^2)^{3/2} dA$ as an integral over $[0, 1] \times [0, 2\pi]$ and evaluate.
3. (Problem 21, p. 392) Let B be the unit sphere. Evaluate

$$\iiint_B \frac{dV}{\sqrt{2 + x^2 + y^2 + z^2}}$$

by making an appropriate change of variables.

4. (Problem 21, p. 392) Using spherical coordinates, compute the integral of $f(\rho, \phi, \theta) = 1/\rho$ over the region in the first octant bounded by the cones $\phi = \pi/4$, $\phi = \tan^{-1} 2$, and the sphere $\rho = \sqrt{6}$.