

## In-class exercises, Feb. 10, 2009

Suppose  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ . The point  $(x_0, y_0)$  is a *critical point* of  $f$  if  $\nabla f(x_0, y_0) = (0, 0)$ . Here is the test for evaluating a critical point to determine whether it is a maximum, minimum, or saddle point for  $f$ . All derivatives are evaluated at  $(x_0, y_0)$ :

- Evaluate the discriminant  $D = f_{xx}f_{yy} - (f_{xy})^2$ .
- If  $D = 0$ , then the test fails.
- If  $D < 0$ , then  $(x_0, y_0)$  is a saddle point.
- If  $D > 0$ , then  $(x_0, y_0)$  is a
  - minimum if  $f_{xx} > 0$
  - maximum if  $f_{xx} < 0$ .

To help check your answers, you can plot the graph  $f$  with Maple as follows. (Select the command-line mode with the [> icon.

```
with(plots);  this only needs to be done once
f := your formula here;
plot3d(f, x = a .. b, y = c .. d);
```

where  $a, b, c, d$  are chosen to bracket the critical point.

## Lab problems

Due at the end of class. Evaluate all the critical points for each of the following functions. Plot each surface to help check your answers.

1. (Problem 1, p. 222)  $f(x, y) = x^2 - y^2 + xy$
2. (Problem 7, p. 222)  $f(x, y) = 3x^2 + 2xy + 2x + y^2 + y + 4$
3. (Problem 17, p. 223)  $f(x, y) = (x^3 + 3y^2)e^{1-x^2-y^2}$
4. (Problem 19, p. 223) This problem illustrates some of the subtleties involved in finding conditions that guarantee that a given function has an extreme value at a given point. Let  $f(x, y) = (y - 3x^2)(y - x^2)$ .
  - (a) Show that the origin is a critical point of  $f$ .
  - (b) Show that  $f$  has a relative minimum at the origin on every straight-line path through the origin. That is, if  $\mathbf{c}(t) = (at, bt)$ , then  $f(\mathbf{c}(t))$  has a relative minimum at  $(0, 0)$  for every choice of  $a$  and  $b$ .
  - (c) Show that the origin is not a relative minimum of  $f$ . (Plot the surface to check.)