In-class exercises, Feb. 3, 2009

Practice problem

(Not to be turned in.) Let $G : \mathbb{R}^2 \to \mathbb{R}$ and suppose we fix $G(x,y) = 0$. This defines y implicitly as a function of x. For example: consider

$$G(x,y) = x^2 + y^2 - \tfrac{1}{2}.$$

If we fix $G(x,y) = 0$, then as x varies, so must y. (Note that fixing $G =$ constant is equivalent to selecting a particular level curve.)

- Describe the graph of $G = 0$.
- The implicitplot command in Maple can plot implicity defined functions. In this case, you can say

  with(plots); *this only needs to be done once*
  implicitplot(x^2+y^2 = 1/2, x = –1 .. 1, y = –1 .. 1);

Lab problems

Due at the end of class.

1. (See Problem 5, p. 159) Use the chain rule to compute $d/dt\, f(\mathbf{c}(t))$ for each of the following functions and paths:

   (a) $\mathbf{c}(t) = (e^t, \cos t)$, $f(x,y) = xy$
   (b) $\mathbf{c}(t) = (t, -t)$, $f(x,y) = x\exp(x^2 + y^2)$

2. Let $G(x,y) = x^2 + y^3 + e^y$. Illustrate an implicitplot command that gives a graph of $G = 0$ in the region $-2 \le x \le 2$, $-2 \le y \le 2$.

3. Suppose $G : \mathbb{R}^2 \to \mathbb{R}$ is a differentiable function. As the previous problem illustrates, the relation $G(x,y) = 0$ defines y implicitly as a function of x. Show that
$$\frac{dy}{dx} = -\frac{\partial G/\partial x}{\partial G/\partial y}$$
   if $\partial G/\partial y \neq 0$. Hint: Consider the "path" $\mathbf{c}(x) = (x, y(x))$ and apply the chain rule to $G(\mathbf{c}(x))$.

4. For the function G defined in Exercise 2, what is dy/dx when $x = -1$?