Answers to in-class exercises, May 4, 2009

1. (Problem 10) See Theorem 8 in §8.3. Since $\mathbf{F}$ is smooth and $\nabla \cdot \mathbf{F} = 2x^2$, the answer is no.

2. (Problem 11) A calculation shows that $\nabla \times \mathbf{F} = 2\mathbf{a}$, which is nonzero in general. Hence $\mathbf{F}$ is not conservative.

3. (Problem 13) The chain rule implies that the value of the line integral is determined solely by the endpoints of the curve C (see the April 29 lecture notes). In this case,

$$\int_C \nabla f \cdot d\mathbf{r} = f(3,0,e^{\pi}) - f(3,0,1) = 0 - 0 = 0.$$

4. The triangle sits in the plane $3x+2y+z=6$, to which a normal is $(3,2,1)$. For this parametrization, the surface area is

$$\int_0^2 \int_0^{3-3x/2} \|(3,2,1)\|\, dy\, dx = 3\sqrt{14},$$

which of course is the area of the triangle.

Remember that a normal vector to the plane $ax+by+cz=d$ is just (a,b,c). For the parametrization $z=(d-ax-by)/c$, we have $\mathbf{T}_x \times \mathbf{T}_y = (a/c, b/c, 1)$.

5. (Problem 19) Use Stokes' theorem. Given $\mathbf{F} = (x+y, 2x-z, y+z)$, we have $\nabla \times \mathbf{F} = (2,0,1)$. A *unit* normal to the plane is $\mathbf{n} = (3,2,1)/\sqrt{14}$, so

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_T (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \iint_T (2,0,1) \cdot \mathbf{n}\, dS = \frac{\sqrt{14}}{2} \times \text{area}(T) = 21,$$

where T is the triangle in the previous problem.