Answers to in-class exercises, March 24, 2009

1. (Problem 13, p. 391) Use cylindrical coordinates:

$$\iint_C ze^{x^2+y^2}\,dV = \int_2^3 \int_0^{2\pi} \int_0^2 zre^{r^2}\,dr\,d\theta\,dz = \tfrac{5}{2}\pi(e^4-1).$$

2. (Problem 14, p. 391) Polar coordinates yield

$$\iint_D (1+x^2+y^2)^{3/2}\,dA = int_0^{2\pi} \int_0^1 r(1+r^2)^{3/2}\,dr\,d\theta = \pi\left(\tfrac{8}{5}\sqrt{2}-\tfrac{2}{5}\right),$$

which follows from the substitution $u = 1+r^2$ in the innermost integral.

3. (Problem 21, p. 392) Spherical coordinates yield

$$\iiint_B \frac{dV}{\sqrt{2+x^2+y^2+z^2}} = \int_0^{2\pi} \int_0^{\pi} \int_0^1 \frac{\rho^2 \sin\phi}{\sqrt{1+\rho^2}}\,d\rho\,d\phi\,d\theta = 2\pi\sqrt{3} - 4\pi \sinh^{-1}\left(\frac{1}{\sqrt{2}}\right).$$

By definition,

$$\sinh^{-1} z = \ln\left(z + \sqrt{1+z^2}\right),$$

so entry 58 in the table of integrals in the back of the book may be expressed equivalently as

$$\int \frac{x^2}{\sqrt{x^2+a^2}}\,dx = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right).$$

4. (Problem 30, p. 392) First note that

$$\sin\left(\tan^{-1} 2\right) = \frac{2}{\sqrt{5}},$$

because $\tan^{-1} 2$ is the angle determined by a right triangle whose adjacent side has length 1 and whose opposite side has length 2 (so the hypotenuse is $\sqrt{5}$). With this in mind, spherical coordinates give

$$\int_0^{2\pi} \int_{\pi/2}^{\tan^{-1} 2} \int_0^{\sqrt{6}} \frac{\rho^2 \sin\phi}{\rho}\,d\rho\,d\phi\,d\theta = 3\pi\left(\frac{\sqrt{2}}{4} - \frac{\sqrt{5}}{10}\right).$$