

## Answers to in-class exercises, Feb. 17, 2009

1. (Problem 5, p. 243) Candidate points for extreme values of  $f$  are defined by the relation  $\nabla f = \lambda \nabla g$ , which gives

$$\begin{aligned}3 &= 4\lambda x \\2 &= 6\lambda y \\3 &= 2x^2 + 3y^2\end{aligned}$$

The first two equations imply  $x = 3/4\lambda$  and  $y = 1/3\lambda$ . The third equation implies  $\lambda = \pm \frac{1}{12}\sqrt{70}$ , so the candidate points at which extrema of  $f$  occur are

$$\mathbf{x}_1 = \left( \frac{-9}{\sqrt{70}}, \frac{-4}{\sqrt{70}} \right) \quad \text{and} \quad \mathbf{x}_2 = \left( \frac{9}{\sqrt{70}}, \frac{4}{\sqrt{70}} \right)$$

A calculation shows  $f(\mathbf{x}_1) = -\frac{1}{2}\sqrt{70}$  and  $f(\mathbf{x}_2) = \frac{1}{2}\sqrt{70}$ .

2. (Problem 12, p. 244) Since the box has no top, its surface area is given by  $SA = w\ell + 2wh + 2h\ell$ . We must maximize  $V = wh\ell$  subject to the constraint  $SA = 16\text{ m}^2$ . The condition  $\nabla V = \lambda \nabla SA$  leads to the system

$$\begin{aligned}h\ell &= \lambda(\ell + 2h) \\wh &= \lambda(w + 2h) \\w\ell &= \lambda(2w + 2h).\end{aligned}$$

Subtracting the second equation from the first leads to the relation  $h(\ell - w) = \lambda(\ell - w)$ , whence  $\ell = w$  or  $\lambda = h$ . The latter implies  $h = 0$ , which is meaningless here.

Substituting  $\ell = w$  into the third equation gives  $w = 4\lambda$ , which, when substituted back into the first equation leads to  $h = w/2$ . Hence  $SA = 16 = 3w^2$ , so  $w = \ell = 4/\sqrt{3}$  m and  $h = 2/\sqrt{3}$  m.

3. (Problem 21, p. 223) We maximize  $V = \ell wh$  subject to the constraint  $\ell + 2w + 2h \leq 108$ . The volume is largest when all three dimensions are largest, so consider the constraint  $g = \ell + 2w + 2h = 108$ . The relation  $\nabla V = \lambda \nabla g$  leads to the system

$$\begin{aligned}wh &= \lambda \\ \ell h &= 2\lambda \\ \ell w &= 2\lambda.\end{aligned}$$

The last two equations imply  $h = w$  and the first two imply  $w = \ell/2$ . The constraint then gives  $w = h = 108/6 = 18$  and  $\ell = 36$ , so the largest-volume box is  $18 \times 18 \times 36 = 11,664 \text{ in}^3$ .