Answers to computer lab exercises, Feb. 3, 2009

1. (See Problem 5, p. 159) Use the chain rule to compute $d/dt\, f(\mathbf{c}(t))$ for each of the following functions and paths:

   (a) Given $\mathbf{c}(t) = (e^t, \cos t)$ and $f(x,y) = xy$, we have

   $$\begin{aligned} \frac{d}{dt} f(\mathbf{c}(t)) &= \nabla f(x,y) \cdot \mathbf{c}'(t) \\ &= (y,x) \cdot (e^t, -\sin t) \\ &= (\cos t, e^t) \cdot (e^t, -\sin t) \\ &= e^t(\cos t - \sin t). \end{aligned}$$

   (b) Given $\mathbf{c}(t) = (t, -t)$ and $f(x,y) = x\exp(x^2+y^2)$, we have

   $$\frac{d}{dt} = e^{x^2+y^2}(1+2x^2, 2xy) \cdot (1,-1) = e^{2t^2}(1+4t^2).$$

2. One possible command is

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   implicitplot(x^2 + y^3 + exp(y) = 0, x = -2 .. 2, y = -2 .. 2);
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3. Given $G(x, y(x)) = 0$, implicit differentiation and the chain rule give

   $$\begin{aligned} 0 &= \frac{d}{dx} G(x, y(x)) \\ &= \nabla G(x,y) \cdot \left(1, \frac{dy}{dx}\right) \\ &= \left(\frac{\partial G}{\partial x}, \frac{\partial G}{\partial y}\right) \cdot \left(1, \frac{dy}{dx}\right). \end{aligned}$$

   Therefore, if $\partial G/\partial y \neq 0$,

   $$\frac{dy}{dx} = -\frac{\partial G/\partial x}{\partial G/\partial y}.$$

4. When $x = -1$, we have

   $$\frac{dy}{dx} = \frac{2}{3y^2 + e^y}.$$

   The solve function on a calculator gives $y \approx -1.1$ when $x = -1$, so $dy/dx \approx 0.5$.