Homework answers, week of April 13, 2009

Section 7.1 (p. 429)

16. (a) For the straight-line path $y = 1 - x$, we have

$$T = \int_0^1 \frac{dx}{\sqrt{2g(1-x)}} = \sqrt{\frac{2}{g}}.$$

(b) We have

$$T = \int_0^1 \frac{dx}{\sqrt{2gy}},$$

where $y = 1 + \sqrt{1-(x-1)^2}$.

Section 7.2 (p. 449)

16. Since $\partial f/\partial x = 2xyze^{x^2}$, we have

$$f = \int 2xyze^{x^2}\,dx = yze^{x^2} + C.$$

Similar calculations for the other two components yield the same thing. We have $C = 5 = f(0,0,0)$, so $f(1,1,2) = 2e + 5$.

Section 7.4 (p. 472)

12. Since $z = \sqrt{x^2+y^2}$ over the unit circle, formula (4) gives

$$A = \iint_D \left[\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 + 1\right]^{1/2} dA = \pi\sqrt{2}.$$

Section 7.5 (p. 481)

10. On the hemisphere of radius R, the mass density is $x^2 + y^2 = R^2\sin^2\phi = R^2(1-\cos^2\phi)$, $0 \le \phi \le \pi/2$. Therefore, the total mass is

$$\int_0^{2\pi}\int_0^{\pi/2} R^2(1-\cos^2\phi)(R^2\sin\phi)\,d\phi\,d\theta = \frac{4R^4\pi}{3}.$$