Homework answers, week of March 23, 2009

Section 4.1 (p. 274)

18. If $\mathbf{c}''(t) = \mathbf{0}$, then each component function $c_i''(t) = 0$. Therefore, $c_i'(t) =$ constant $= a_i$, so another integration yields $c_i(t) = a_i t + b_i$ for some constant b_i. In other words, $\mathbf{c}(t) = \mathbf{a}t + \mathbf{b}$ for constant vectors $\mathbf{a}$ and $\mathbf{b}$. If $\mathbf{a} \neq \mathbf{0}$, then $\mathbf{c}$ is a line; otherwise, it is a point.

Section 4.3 (p. 294)

17. A satellite in an orbit just about the earth's surface is about R_0 away from the earth's center of gravity. From the discussion leading to Kepler's law on p. 266, the speed s of the satellite is

$$s = \sqrt{\frac{GM}{R_0}} = \sqrt{gR_0}.$$

Hence the kinetic energy of the satellite is $\frac{1}{2}mv^2 = \frac{1}{2}mgR_0$. The escape velocity $v_e = \sqrt{2gR_0}$, so the corresponding kinetic energy is twice as large.

Section 4.2 (p. 312)

28. $\mathbf{F}+\mathbf{G}$ must have zero divergence, because

$$\begin{aligned}
\nabla\cdot(\mathbf{F}+\mathbf{G}) &= (\partial_x, \partial_y, \partial_z)\cdot(F_1+G_1, F_2+G_2, F_3+G_3) \\
&= \partial_x(F_1+G_1)+\partial_y(F_2+G_2)+\partial_z(F_3+G_3) \\
&= (\partial_x F_1+\partial_y F_2+\partial_z F_3)+(\partial_x G_1+\partial_y G_2+\partial_z G_3) \\
&= \nabla\cdot\mathbf{F}+\nabla\cdot\mathbf{G} \\
&= 0+0.
\end{aligned}$$