# Selected answers to homework, week of Jan. 26

**Page 107, Problem 32.** In polar coordinates, we have

$$f(r,\theta) = \frac{2r^2\cos\theta\sin\theta}{r^2} = 2\cos\theta\sin\theta = \sin 2\theta.$$

Hence the level curves are rays extending outward at constant angles $\theta$. The origin is excluded; its value is 0 regardless of $\theta$.

**Page 125, Problem 6.** (a) 1; the function is continuous so simply substitute to find the obvious limit.

(b) Use l'Hôpital's rule:

$$\lim_{x\to 0}\frac{\sin^2 x}{x} = \lim_{x\to 0}\frac{2\sin x\cos x}{1} = \lim_{x\to 0}\sin 2x = 0.$$

(c) Use l'Hôpital's rule:

$$\lim_{x\to 0}\frac{\sin^2 x}{x^2} = \lim_{x\to 0}\frac{2\sin x\cos x}{2x} = \lim_{x\to 0}\frac{\sin 2x}{2x} = \lim_{x\to 0}\frac{2\sin x}{2} = 1.$$

**Page 126, Problem 12.** (a) Use l'Hôpital's rule:

$$\begin{aligned}\lim_{x\to 0}\frac{\sin 2x - 2x}{x^3} &= \lim_{x\to 0}\frac{2\cos 2x - 2}{3x^2}\\ &= \lim_{x\to 0}\frac{-4\sin 2x}{6x}\\ &= \lim_{x\to 0}\frac{-8\cos 2x}{6}\\ &= -\tfrac{4}{3}.\end{aligned}$$

(b) Suppose for example that we approach the origin along the $x$ axis ($y$ held constant at 0). Then, by part (a), the limiting value is $-4/3$. If we then approach the origin along the $y$ axis ($x$ held constant at 0), then the ratio is constant at 1. Because the limiting values differ depending on the path to the origin, the limit does not exist.

(c) Use polar coordinates:

$$\begin{aligned}
\lim_{(x,y,z)\to(0,0,0)} \frac{2x^2 y\cos z}{x^2+y^2} &= \lim_{(r,\theta,z)\to(0,0,0)} \frac{2r^3\cos^2\theta\sin\theta\cos z}{r^2} \\
&= \lim_{(r,\theta,z)\to(0,0,0)} 2r\cos^2\theta\sin\theta\cos z \\
&= 0.
\end{aligned}$$