

1) Given the following transition matrix;

$$\begin{array}{c} A \quad B \quad C \quad D \quad E \\ \begin{array}{l} A \\ B \\ C \\ D \\ E \end{array} \begin{bmatrix} .5 & .4 & 0 & .1 & 0 \\ .3 & .3 & 0 & .2 & .2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ .4 & .3 & .2 & 0 & .1 \end{bmatrix} \end{array}$$

- Which state(s) are absorbing?
- If we start in state A, for how many steps should we expect to be in state A? (4 decimal places)
- If we start in state E, for how many steps should we expect to be in state B? (4 decimal places)
- If we start in state B, how many steps should we expect before absorption? (4 decimal places)
- If we start in state A, how many steps should we expect before absorption? (4 decimal places)
- If we start in state B, Find the probability that we are eventually absorbed into state C. (4 decimal places)
- If we start in state E, Find the probability that we are eventually absorbed into state C. (4 decimal places)

2) Given the following transition matrix;

$$\begin{array}{c} A \quad B \quad C \quad D \quad E \\ \begin{array}{l} A \\ B \\ C \\ D \\ E \end{array} \begin{bmatrix} .2 & .3 & 0 & .2 & .3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & .3 & .5 & .1 & .1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

- Which state(s) are absorbing?
- If we start in state A, for how many steps should we expect to be in state B? (4 decimal places)
- If we start in state C, for how many steps should we expect to be in state A? (4 decimal places)
- If we start in state A, how many steps should we expect before absorption? (4 decimal places)
- If we start in state C, how many steps should we expect before absorption? (4 decimal places)
- If we start in state A, Find the probability that we are eventually absorbed into state D. (4 decimal places)
- If we start in state A, Find the probability that we are eventually absorbed into state E. (4 decimal places)

3) Penelope needs \$5. She will keep betting \$1 until either reaching \$5 or going broke. For each bet, the probability of winning is 40% (no ties).

- Find the transition matrix.
- If she starts with \$2, how many times will she have \$3 (4 decimal places)
- If she starts with \$3, how many times will she play before being absorbed (winning or losing) ?(4 decimal places)
- If she starts with \$1, find the probability of Penelope getting to \$5. (4 decimal places)

4) Repeat problem 3, but with the probability of a win at 40% and the probability of a tie at 15%. In the case of a tie, no money is gained or lost.

a) Find the transition matrix.

b) If she starts with \$2, how many times will she have \$3 (4 decimal places)

c) If she starts with \$3, how many times will she play before being absorbed (winning or losing) ?(4 decimal places)

d) If she starts with \$1, find the probability of Penelope getting to \$5. (4 decimal places)

5) Given the following zero-sum games; A: $\begin{bmatrix} 8 & 3 \\ 7 & 4 \\ 4 & 5 \end{bmatrix}$ B: $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 2 & 5 & -6 \end{bmatrix}$ C: $\begin{bmatrix} 1 & -5 & 2 \\ 6 & -6 & 10 \\ 0 & -8 & 5 \end{bmatrix}$

a) Is game A strictly defined. (YES or NO)

b) If game A is strictly defined, find the value. (If not, leave this blank)

c) Is game B strictly defined. (YES or NO)

d) If game B is strictly defined, find the value. (If not, leave this blank)

e) Is game C strictly defined. (YES or NO)

f) If game C is strictly defined, find the value. (If not, leave this blank)

g) Is game C in favor of player I or player II (Player I plays rows).

Solutions to Practice 9 (revision 0)

1a) C and D b) 3.9310 c) 2.1379 d) 6.2069 e) 6.9655 f) .1379 g) .3172

2a) D and E b) .6 c) 1.2 d) 2.6 e) 4.16 f) .4 g) .6

$$3a) P = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ .6 & 0 & .4 & 0 & 0 & 0 \\ 0 & .6 & 0 & .4 & 0 & 0 \\ 0 & 0 & .6 & 0 & .4 & 0 \\ 0 & 0 & 0 & .6 & 0 & .4 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$\text{subdivided } P = \begin{matrix} & 0 & 5 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ 5 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ .6 & 0 & 0 & .4 & 0 & 0 \\ 0 & 0 & .6 & 0 & .4 & 0 \\ 0 & 0 & 0 & .6 & .4 & 0 \\ 0 & .4 & 0 & 0 & .6 & 0 \end{bmatrix} \end{matrix}$$

b) 1.1848 c) 5.9953 d) .0758

$$4a) P = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ .45 & .15 & .4 & 0 & 0 & 0 \\ 0 & .45 & .15 & .4 & 0 & 0 \\ 0 & 0 & .45 & .15 & .4 & 0 \\ 0 & 0 & 0 & .45 & .15 & .4 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$\text{subdivided } P = \begin{matrix} & 0 & 5 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ 5 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ .45 & 0 & .15 & .4 & 0 & 0 \\ 0 & 0 & .45 & .15 & .4 & 0 \\ 0 & 0 & 0 & .45 & .15 & .4 \\ 0 & .4 & 0 & 0 & .45 & .15 \end{bmatrix} \end{matrix}$$

b) 1.7594 c) 7.1557 d) .1559

5a) no c) no e) yes f) -5 g) Player II