8.3 Quick Notes

Expected Value

If a sample space S is partitioned into events A_1 through A_n (where $A_1 \cup A_2 \cup \ldots\ldots \cup A_n = S$),
with respective payoffs x_1 through x_n (x_i is A_i's payoff) and $p_i = P(A_i)$, i=1,2,.....n)

then the The Expected Payoff is given by

$$E = x_1 p_1 + x_2 p_2 + \ldots\ldots + x_n p_n = \sum_{i=1}^{n} \left[x_i P(A_i) \right]$$

For the Binomial distribution, the expected number of successes in n Bernoulli trials can be computed simply as

E=np

A game is **fair** if its expected payoff is 0.

Examples.

1) In a scratch off game, the probability of winning \$100 is 0.1%, the probability of winning \$50 is 0.3%, the probability of winning \$25 is 1%, and the probability of winning \$10 is 3%. (Otherwise you don't win any money)

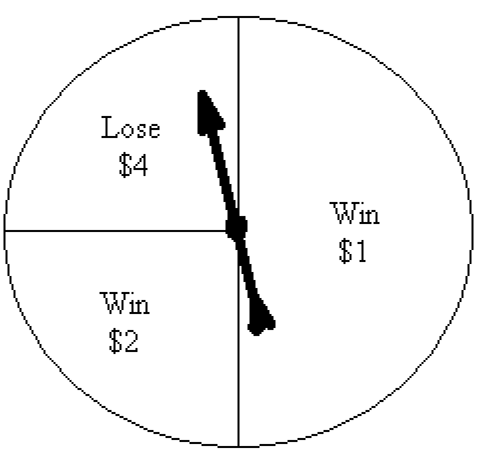
a) Find the game's expected payoff.

b) Is this a fair game? If not, how could we make it a fair game?

c) If the game cost \$1 to play, is it in your favor? What would the expected payoff be?

2) It cost \$2 to play a dice game. Two dice are rolled. If the sum is at least 10, then you are given \$12, otherwise you are given nothing. Is this a fair game?

3) A gambling game is played as follows, the spinner below is spun, then a die is rolled. If the result on the die is odd, then that amount in dollars is subtracted from the amount on the spinner, if it is even, it is added.



a) Find the expected value of the game.

b) Is the game in the player's favor?