

## 2.6

## Quick Notes

### Inverse Matrices (with the calculator)

1)  $A^{-1}$  is the inverse matrix of  $A$  if and only if  $A$  is a square matrix,  $A^{-1}A = I$ , and  $AA^{-1} = I$ .

Example (Ti – 83)

2) To find the inverse of the matrix  $\begin{bmatrix} 2 & 5 \\ 7 & -3 \end{bmatrix}$ , first, enter the matrix into the calculator under  $A$ . (quit out of the screen when finished). Then Hit (2<sup>nd</sup>)  $MATRX$ , then  $A$ , then  $x^{-1}$ , then enter.

Converting to fractions, you will get  $\begin{bmatrix} \frac{3}{41} & \frac{5}{41} \\ \frac{7}{41} & \frac{-2}{41} \end{bmatrix}$ . If a matrix is not invertible (doesn't have an inverse, then your

calculator will give an error (double check your work).

Ex) Find the inverse of each matrix.

a)  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$     b)  $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 0 & -1 \\ 7 & 3 & 2 \end{bmatrix}$     c)  $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & -1 \\ 4 & 3 & 2 \end{bmatrix}$

3) Application: solving a consistent system of  $n$  linear equations with  $n$  variables (with one solution)  $AX = B$

( $A$  will be an  $n \times n$  matrix, and  $X$  and  $B$  will be  $n \times 1$ )

To solve a system $AX = B$	1) Multiply both sides by $A^{-1}$	$A^{-1}AX = A^{-1}B$
	2) $A^{-1}A = I$	$IX = A^{-1}B$
	3) $IX = X$	$X = A^{-1}B$

The matrix  $X$  will show the solution.

Application example. Given the following system of equations,

$$3x + 2y - 7z = 35$$

$$3x + 4z = 7$$

$$2x - y - z = 9$$

1) Write the system in matrix form ( $AX = B$ )

2) Solve the system.

## Application – Encryption & Decryption

Using the simple code

A = 1	E = 5	I = 9	M = 13	Q = 17	U = 21	Y = 25
B = 2	F = 6	J = 10	N = 14	R = 18	V = 22	Z = 26
C = 3	G = 7	K = 11	O = 15	S = 19	W = 23	
D = 4	H = 8	L = 12	P = 16	T = 20	X = 24	

1) Let's encrypt the phrase "SHOOTING STAR" using the following matrices.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

a) First with matrix A. Since A is a  $2 \times 2$  matrix, we will be dividing our phrase into two letter strings; SH OO TI NG ST AR. From the simple code above, S = 19 and H = 8, so we will turn this into the matrix

$$\begin{bmatrix} 19 \\ 8 \end{bmatrix}.$$

Multiplying matrix A times  $\begin{bmatrix} 19 \\ 8 \end{bmatrix}$ , we get  $\begin{bmatrix} 51 \\ 62 \end{bmatrix}$ , thus we have code "51 62" for SH. Doing the same for the each pair of letters, we finish with the code "51 62 75 75 56 67 42 49 99 98 73 56". (Note that in this case, both O's are represented by 75, but that is not always the case. Notice that the first S and the second S are different)

b) Now with matrix B. Since B is a  $3 \times 3$  matrix, we will be dividing our phrase into three letter strings; SHO OTI NGS TAR. From the simple code above, S = 19, H = 8 and O = 15, so we will turn this into the

matrix  $\begin{bmatrix} 19 \\ 8 \\ 15 \end{bmatrix}$ . Multiplying matrix B times  $\begin{bmatrix} 19 \\ 8 \\ 15 \end{bmatrix}$ , we get  $\begin{bmatrix} 29 \\ 78 \\ 45 \end{bmatrix}$ , thus we have code "51 62 45" for SHO. Doing

the same for the remaining three letter strings, we finish with "29 78 45 33 94 37 52 75 78 56 80 93".

2) To decrypt we use the inverse matrices.

a) To use matrix A, we will decrypt 2 numbers at a time (since it is a  $2 \times 2$ ) To decrypt the first two numbers

from part 1a, we multiply  $A^{-1}$  times  $\begin{bmatrix} 51 \\ 62 \end{bmatrix}$ , which will give us  $\begin{bmatrix} 19 \\ 8 \end{bmatrix}$ . 19 represents S and 8 represents H. After

doing them all, we get "SHOOTINGSTAR". Using this technique, it will be up to the end user to decide where the spaces belong.

b) To use matrix B, we will decrypt 3 numbers at a time (since it is a  $3 \times 3$ ). Multiply  $B^{-1}$  times  $\begin{bmatrix} 29 \\ 78 \\ 45 \end{bmatrix}$  and so on.

(Note that, with the technique we are using in class, shooting star has 12 letters, and the dimension of the key square matrix must go into 12. Discuss other techniques to avoid this problem in class, such as adding code 27 for a space)