2.5 Quick Notes

Matrix Multiplication

If A and B are matrices, in order to perform the operation AB (A times B), the number of columns in A must equal the number of rows in B. Otherwise, it is not possible. If matrix A has dimensions $m \times k$ and matrix B has dimensions $k \times n$, then AB will have dimensions $m \times n$.

Matrix multiplication is not commutative. In general, AB doesn't equal BA!!!!!

Dot products - A vector can be considered to be an $1 \times n$ matrix (row vector) or a $n \times 1$ matrix (column vector). Both of these would have n dimensions. If v and w are vectors with the same dimensions, so if $v = \langle a_1, a_2, a_3, \ldots\ldots a_n \rangle$ and $w = \langle b_1, b_2, b_3, \ldots\ldots b_n \rangle$ then the dot product ("v dot w") would be as follows;

$$v \cdot w = a_1b_1 + a_2b_2 + a_3b_3 + \ldots\ldots + a_nb_n$$

How to multiply two matrices - If the number of columns in matrix A equals the number of rows in matrix B, then we can multiply AB to get matrix C. To calculate each individual entry, c_{ij}, we simply take the dot product of the i^{th} row of A and the j^{th} column of B.

Examples. Given the following matrices

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 \\ 3 & 0 \\ 4 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 3 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

Find the following, if possible. a) AB b) BA c) AC

d) Find the missing variable $\begin{bmatrix} 3 & x \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ x & 6 \end{bmatrix} = \begin{bmatrix} 22 & 39 \\ 0 & x \end{bmatrix}$

Identity Matrix – a square matrix C in which $c_{ij} = 1$ if i = j, and 0 if i j.

examples $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

If A is an $m \times n$ matrix, then $AI_n = A$ and $I_mA = A$.

Properties – If A is an $m \times r$ matrix, B is an $r \times k$ matrix, and C is a $k \times n$ matrix, then A(BC) = (AB)C
If A is an $m \times k$ matrix, and B and C are both $k \times n$ matrices, then A(B + C) = AB + AC

Examples. Use the matrices above to find the following, if possible.

a) B(2A – 3C) b) ABC c) CBA d) ACB e) BCA