

10.6

Quick Notes

Optimal Strategies for a 2×2 zero sum game

1) If a 2×2 zero sum game defined by the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is **not strictly determined**, then...

The optimal strategy for player I is $P = [p_1 \quad p_2]$

$$\text{where } p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \quad \text{and} \quad p_2 = \frac{a_{11} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

And the optimal strategy for player II is $Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$

$$\text{where } q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \quad \text{and} \quad q_2 = \frac{a_{11} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

Examples)

1) Find the optimal strategies and the expected payoffs for each of the following two person, zero sum games.

a) $\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 4 & -3 \\ 5 & -6 \end{bmatrix}$

c) $\begin{bmatrix} 4 & -3 \\ -7 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 4 & -3 \\ 2 & 0 \end{bmatrix}$