10.5 Quick Notes

Mixed Strategies

1) Given the zero sum game that is not strictly determined: A= $\begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix}$, player I can always play row 1,since he can then win \$5, but if he does, player II will catch on sooner or later and then he will select column 2.

2) So player I may choose to "mix it up". Let's say that player I plays row 1 60% of the time, and row 2 40% of the time. Also, let's say that player II plays column 1 30% of the time, and column 2 70% of the time.
If the players play their strategies randomly, then the expected payoff would be :
(.60)(.30)(5) + (.60)(.70)(-1) + (.40)(.30)(-2) + (.40)(.70)(1) = .52 which means that every time the game is played, on average, player II will pay player I .52.

3) **Matrix formula for expected payoff.**

Let A be the matrix representing a two person, zero sum game
Let P be a row vector representing Player I's strategy (probability distribution)
Let Q be a column vector representing Player II's strategy (probability distribution)

If the players play their strategies **randomly**,

then the expected payoff of the two person zero sum game would be E = PAQ

From the above example: $E = \begin{bmatrix} .60 & .40 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} .30 \\ .70 \end{bmatrix} = \begin{bmatrix} .52 \end{bmatrix}$

(BOARDWORK)

1) Consider the zero sum game $\begin{bmatrix} 4 & -3 \\ -7 & 6 \end{bmatrix}$

a) Each player plays each of their options with equal probabilities, then find the expected payoff.
b) If player I plays row 1 10% of the time, and row 2 90% of the time, and player II plays column 1 35% of the time, and column 2 65% of the time, then find the expected payoff.
c) If player I plays row 2 twice as often as row 1, and player II plays column 1 three times as often as column 2, then find the expected payoff.