

10.3

Quick Notes

Absorbing Markov Chains

- 1) A state is called an absorbing state if, once you go into the state, you can't leave. (If the transition matrix' columns and rows are in order by state, then $P_{ii} = 1$)
- 2) An absorbing Markov chain is one that contains an absorbing state, and it is possible to get to an absorbing state from each a nonabsorbing state, if not directly than indirectly.

Ex) Which of the following would be a transition matrix from an absorbing Markov chain?

a) $\begin{bmatrix} 1 & 0 \\ .5 & .5 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} .4 & .6 & 0 \\ .3 & .7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & .6 & .4 \\ .3 & .7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3) By rearranging the states in the transition matrix P (absorbing states first, then nonabsorbing), and thus

rearranging the entries, we can subdivide P into the following; $P = \begin{bmatrix} I_r & 0 \\ S & Q \end{bmatrix}$ I_r is an $r \times r$ identity matrix, 0 is a matrix of all zeros. S and Q are matrices will be used in calculations to follow.

4) The fundamental matrix of an absorbing Markov chain (T), is found as follows; $T = [I - Q]^{-1}$

The entries of T give the expected number of times that the process is in each nonabsorbing state, provided that it began in a nonabsorbing state. (t_{ij} gives the expected number of times the process is in state j, if it started in state i)

5) If we start in nonabsorbing state i, then the total expected number of steps before absorption can be found by adding up all of the entries in state i's row of T.

6) The ij^{th} entry in the matrix TS gives the probability that if we start in the nonabsorbing state I, we end up in the absorbing state j.

Examples:

S1 S2 S3 S4 S5

1) $\begin{matrix} S1 \\ S2 \\ S3 \\ S4 \\ S5 \end{matrix} \begin{bmatrix} .3 & .1 & .6 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ .4 & 0 & 0 & .1 & .5 \\ 0 & 0 & 0 & 1 & 0 \\ .3 & 0 & .2 & .1 & .4 \end{bmatrix}$

- a) If we start in state 1, for how many steps can we expect to be in state 3 before we are absorbed?
- b) If we start in state 3, how many steps can we expect to go before we are absorbed?
- c) If we start in state 1, find the probability that we are absorbed into state 2.
- d) If we start in state 3, find the probability that we are absorbed into state 2.

2) Herbert needs 6 cents for a piece of candy. He will keep betting 2 cents until he either goes broke or has enough for the candy. In each bet, he has a 40% chance of winning the bet (gain 2 cents) and a 60% chance of losing the bet (lose 2 cents). Find the probability that he gets the candy if he starts with;

- a) 2 cents b) 4 cents