

10.2

Quick Notes

Regular Markov Chains

- 1) If P is a transition matrix, then P^n gives the probability of moving from state i to state j in n steps (observations, stages).
- 2) A Markov Chain is regular if, for some (any) power of the transition matrix P , all entries are positive (no 0's)
(Note: if P^n has all positive entries, the every consecutive power of P after that will too)
- 3) Long term behavior of a regular Markov chain.
 - a) As n gets large, P^n approaches a fixed matrix T ($P^n \rightarrow T$)
 - b) The rows of T are identical (row probability vector t)
 - c) $tP = t$
 - d) if $v^{(0)}$ is the initial probability distribution, then as n gets large, $v^{(n)} \rightarrow t$

Examples.

- 1) If a Markov chain has the following transition matrix, would it be regular?

a) $\begin{bmatrix} 1 & 0 \\ .5 & .5 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ .5 & .5 \end{bmatrix}$ c) $\begin{bmatrix} .9 & .1 \\ .5 & .5 \end{bmatrix}$

- 2) There are two insurance companies in town; BigMega inc. and Mom's insurance. All people in town have one or the other. Every year, 5% of BigMega's customers switch to Mom's (the others stay), and 3% of Mom's customers move to BigMega (the others stay).
 - a) Find the initial probability distribution.
 - b) Find the transition matrix.
 - c) Eventually, in the long run, what percentage of the customers will each have?