

# 10.1

# Quick Notes

## Markov Chains

1) Markov Chain – Sequence of experiments, each of which results in one of a finite number of states (1, 2, 3, ...m)

2) A Transition Matrix (P) is a square matrix whose entries represent the probability of moving from one state to another.  $p_{ij}$  represents the probability of moving from state  $i$  to state  $j$  in one observation (step).

s1    s2

For example, if  $P = \begin{matrix} & \begin{matrix} s1 & s2 \end{matrix} \\ \begin{matrix} s1 \\ s2 \end{matrix} & \begin{bmatrix} .85 & .15 \\ .45 & .55 \end{bmatrix} \end{matrix}$ , then if we start in state 1 (s1), after 1 step, the probability of staying in s1 is .

85, and the probability of moving to s2 is .15. If we start in state 2 (s2) then, after step, the probability of staying there is .55, and the probability of moving to s1 is .45.

Each row in a transition matrix is a probability distribution, so the sum of the entries in the row is 1.

3) The initial probability distribution ( $v^{(0)}$ ) is a  $1 \times n$  row vector (if there are  $n$  states) whose entries represent the probability of starting in each respective state. For example, if there were two states, and we had an equal chance of starting in either one, then the initial probability distribution would be  $v^{(0)} = \langle .5 \ .5 \rangle$ . If we were guaranteed to start in state 1, it would be  $v^{(0)} = \langle 1 \ 0 \rangle$ . If we were guaranteed to start in state 2, it would be  $v^{(0)} = \langle 0 \ 1 \rangle$ .

The probability distribution after  $k$  steps (observations, stages) is denoted as  $v^{(k)}$ , and can be calculated as follows;

$$v^{(k)} = v^{(k-1)}P \text{ or } v^{(k)} = v^{(0)}P^k \quad \text{where } P \text{ is the transition Matrix.}$$

Examples.

1) There are two insurance companies in town; BigMega inc. and Mom's insurance. All people in town have one or the other. Every year, 5% of BigMega's customers switch to Mom's (the others stay), and 3% of Mom's customers move to BigMega (the others stay).

a) If, right now, BigMega has 94% of the customers in town, and Mom's has the rest, find the initial probability distribution.

b) Find the transition matrix.

c) After 1 year, what percentage of the customers will each have?

d) after 5 years, what percentage will each have?

2) (In class) Mouse in rooms example.