STP 226
ELEMENTARY STATISTICS

CHAPTER 5

Probability Theory - science of uncertainty

5.1 Probability Basics

Equal-Likelihood Model

Suppose an experiment has N possible outcomes, all **equally likely.** Then the probability that a specified event occurs equals the number of ways, *f*, that the event can occur, divided by the total number of possible outcomes, *N.*

Probability of an event = *f/N*

Example: Five top Oklahoma State officials are:
Governor (G), Lieutenant Governor (L) Secretary of State (S), Attorney General (A) and Treasurer (T).
Suppose we take a simple random sample without replacement of 3 officials from above 5, what is the probability that the sample will include Treasurer?

Solution: The following are all possible samples of 3 officials:
GLS, GLA, GLT, GSA, GST, GAT, LSA, LST, LAT, SAT
Out of 10 of them, 6 include Treasurer (T), so probability of obtaining a sample that include a Treasurer is 60%.

The essential idea in the above example was that when outcomes are equally likely, probabilities are simply percentages (relative frequencies)

Example: Family is defined to be a group of two or more people related by birth, marriage or adoption and residing together in a household. The following table gives the distribution of U.S families according to the size, the frequencies are in thousands:

Size	Frequency
2	33,706
3	16,652
4	15,149
5	6,619
6	2,298
7+	1,173

Suppose a U.S family is randomly selected, find probability that:

a. Family size is at most 3 people
b. Family size is between 4 and 5 people inclusive
c. Family size is 1 person
d. Family size is at least 2 people

Solution: Since total number of families is 75597 thousands,

answer for (a) is (33706+16652)/75597=.666 or 66.6%
answer for (b) is (15149+6619)/75597=.288 or 28.8%
answer for (c) is 0/75597=0 or 0%, impossible event
answer for (d) is 75597/75597=1.0 or 100%, certain event

Again we used the fact that all selections are equally likely and in each case answer was a relative frequency of a particular outcome.

Experiment - an action (experiment) with chance outcomes.

Event - some specified result that may or may not occur when the experiment is performed.

The Meaning of Probability

- generalization of the concept of percentage, for example if we select a random member of a finite population, probability that the member will have a particular characteristics is the percentage of the population with that characteristics.
  EX: Population of certain small town has 40% of Democrats, 50% of Republicans, 10% Libertarian registered voters. If we randomly select a registered voter from that town, probability of this voter to be Libertarian is 10%.
- probability near 0 indicates that an event is very unlikely to occur, impossible event has probability 0 of occurring.
- probability near 1 (100%) suggest that an event is quite likely to occur, 100% is the probability of certain event.

Frequentist interpretation of probability – Probability of an event is a proportion of times it occurs in a large number of repetitions of the experiment.

Ex. If balanced coin is tossed 750 times and head occurs 377 times, probability of head showing can be estimated as 377/750=.503

Ex. Probability of a girl being born in some Phoenix hospital is .512. Frequentist would explain: it means that in 1000 births observed in that hospital 512 were girls.

Probability model - a mathematical description of the experiment based on certain primary aspects and assumptions.

- ex. **Equal-likelihood model** (assume that all possible outcomes are equally likely to occur). This will be the model we will consider, but there are other models that do not have the similar assumptions.

Basic Properties of Probabilities

1. The probability of an event is always between 0 and 1, inclusive.
2. The property of an event that cannot occur is 0 (impossible event).
3. The property of an event that must occur is 1 (certain event).

5.2 Events

Sample space - The collection of all possible outcomes for an experiment. We will consider finite sample spaces, where we can list or at least count the number of all possible outcomes.

Event - collection of outcomes for the experiment, that is, any subset of the sample space.

Example1: Experiment: Select a card from an ordinary deck.
Sample Space: Set of all 52 cards
A=event that a card is a spade= set of all 13 spades.

Example2: Experiment: Roll two balanced dice.
Sample space = set of 36 pairs of numbers like (1,1), (1,3) , (3, 1) (2, 2).....
A=event that a sum of both dice is 4 = { (2,2), (3, 1), (1, 3) }

Example3: Experiment: Select a random student from the large University with 40% Democrats, 50% of Republicans and 10% of other party affiliation.
Sample space: All students at that University
A=event that a student is a Democrat=set of all students at that University that are Democrats

Relationship among events can be represented using **Venn diagrams.**
E - an event.

$\bar{E}$ = **(not E)** - the event that E does not occur (complement of E), consists of outcomes in the sample space that are not in E

$A \cap B$ **= (A & B)** - the event that both A and B occur, consists of outcomes that are common to both events.

$A \cup B$ = **(A or B)** - the event that either A or B or both occur, consists of outcomes that are in A, B or both.

Mutually Exclusive Events

Two or more events are said to be **mutually exclusive** if at most one of them can occur when the experiment is performed, that is, if no two of them have outcomes in common. In other words none of them share any outcomes.

5.3 Some Rules of Probability

Probability Notation

If E is an event, then **P(E)** stands for the probability that event E occurs. It is read "the probability of E."

Going back to the examples from section 5.2
In Example 1 P(A)=13/52=1/4
In Example 2 P(A)=3/36=1/12
In Example 3 P(A)=.40

The Special Addition Rule

If event A and event B are mutually exclusive, then
P(A or B) = P(A) + P(B).

$$P(A\cup B)=P(A)+P(B)$$

More generally, if events A, B, C, are mutually exclusive, then

P(A or B or C) = P(A) + P(B) + P(C)

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)$$

In words, for mutually exclusive events, the probability that one or another of the events occurs equals the sum of the individual probabilities.

Example: If we randomly select a card from an ordinary deck, Let A= event that card is Queen, B = event that card is Jack, C= event that card is “10”. Probability that randomly selected card is a Queen, Jack or “10” is

$P(A\cup B\cup C)=\frac{4}{52}+\frac{4}{52}+\frac{4}{52}=\frac{12}{52}=\frac{3}{13}$, since events are not overlapping.

The Complement Rule

For any event E, P(not E) +P(E)=1 , so
P(not E)= 1 – P(E) and P(E)= 1-P(not E)

$P(\bar{E})=1-P(E)$ and $P(E)=1-P(\bar{E})$

In words, the probability that an event does not occur equals 1 minus the probability that it does occur and probability that event occurs is equal 1 minus probability that it does not occur.

Example: If we roll a balanced die 2 times and
A=event that sum of both tosses is not 4 , then
event not A= event that sum of both tosses is 4
not A={(2,2), (1,3), (3,1)}

P(not A)=3/36, P(A)=1-3/36=33/36

The General Addition Rule

If A and B are any two events, then
P(A or B) = P(A) + P(B) - P(A & B).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

In words, for any two events, the probability that one or the other occurs equals the sum of the individual probabilities less the probability that both occur.

Example: If a card is selected from an ordinary deck and A= event that card is a King, B=event that card is red, then probability that card is a King or red is given by:
P(A or B)=4/52+26/52 -2/52 = 28/52=7/13(2 of Kings are also red cards)