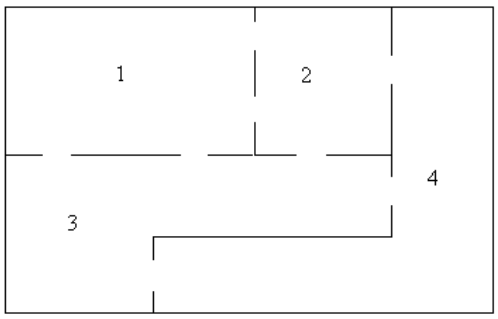
MAT 119 10.1-10.2 Practice 8

1) Given the following transition matrix;

$$P = \begin{array}{c} \\ s1 \\ s2 \end{array} \begin{array}{c} \begin{array}{cc} s1 & s2 \end{array} \\ \begin{bmatrix} .3 & .7 \\ .1 & .9 \end{bmatrix} \end{array}$$

a) Find the probability of moving from state 1 to state 2 in one observation.
b) Find the probability of moving from state 1 to state 2 in three observations.
c) Find the initial probability distribution if the system is initially in state 1.
d) Find the initial probability distribution if the system is twice as likely to start in state 2 as it is to start in state 1.
e) If the system is initially in state 2, then find the probability distribution after 4 observations.
f) Find the fixed probability vector (t)
g) Eventually, find the probability of being in state 1.
h) Eventually, find the probability of being in state 2.



2) If a mouse is placed into the enclosure to the left, it moves as follows; If the mouse is in any particular room, it can either stay in the room or go out of any of the room's doors, each option having the same probability. For example, if it is in room 2, it is more likely to go to room 1 than anything else because there are two doors.

a) Find the transition matrix that goes with this system.
b) If the mouse is in room 1,find the probability that it moves to room 3 in the next observation?
c) If the mouse is in room 3,find the probability that it moves to room 4 in the next observation?
d) If the mouse starts in room 3, which room will it most likely be in after four observations?
e) Find the probability of it being in that room from part d (4 decimal places).
f) If the mouse starts in room 3, in which room is it most likely to be after eight observations?
g) Find the probability of it being in that room from part f (4 decimal places).

3) Using the same enclosure from problem 2, If a mouse is placed into the enclosure, it moves as follows;
If the mouse is in any particular room, it will leave the room in the next observation by going out of any of the room's doors, each door having the same probability.
a) Find the transition matrix that goes with this system.
b) If the mouse is in room 1,find the probability that it moves to room 3 in the next observation?
c) If the mouse is in room 3,find the probability that it moves to room 4 in the next observation?
d) If the mouse starts in room 2, which room will it most likely be in after three observations?
e) Find the probability of it being in that room from part d (4 decimal places).
f) If the mouse starts in room 2, in which room is it most likely to be after eight observations?
g) Find the probability of it being in that room from part f (4 decimal places).

4) Is the Markov chain whose transition matrix is given regular?

a) $\begin{bmatrix} 1 & 0 \\ .2 & .8 \end{bmatrix}$ b) $\begin{bmatrix} .8 & .2 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} .3 & .7 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ .5 & .5 \end{bmatrix}$

5) There are two gas stations in town; Barny's and Auntie M's. In town each year, 17% of Barny's customers switch to Auntie M's, and 8% of Auntie M's customers switch to Barny's.(Let Barny's be state 1 and Auntie M's be state 2)
a) Find the transition matrix for the system.
b) Find the fixed probability vector (t) (fraction form)
c) What portion of the area's market will Barny's eventually hold? (fraction form)
d) What portion of the area's market will Auntie M's eventually hold? (fraction form)

Solutions to Practice 8 (revision 0)

1a) .7 b) .868 c) < 1, 0 > d) $\langle \frac{1}{3}, \frac{2}{3} \rangle$ e) < .1248, .8752 > f) $\langle \frac{1}{8}, \frac{7}{8} \rangle$ g) $\frac{1}{8}$ h) $\frac{7}{8}$

2a) $\begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ (the rows on the right and the columns on top represent rooms 1 through 4 in order)

b) $\frac{2}{5}$ c) $\frac{1}{3}$ d) Room 3 e) .3103 f) Room 3 g) .3003

3a) $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{2}{5} & \frac{1}{5} & 0 & \frac{2}{5} \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$ (the rows on the right and the columns on top represent rooms 1 through 4 in order)

b) $\frac{1}{2}$ c) $\frac{2}{5}$ d) Room 1 e) .3583 f) Room 3 g) .3446

4a) no b) no c) yes d) yes

5a) $\begin{bmatrix} .83 & .17 \\ .08 & .92 \end{bmatrix}$ b) < .32, .68 > c) 32% d) 68%