Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 176 (2007) 264–274
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Bucket brigades with worker learning

Dieter Armbruster [a], Esma S. Gel [b,*], Junko Murakami [a]

[a] *Department of Mathematics, Arizona State University, Tempe, AZ 85287, USA*
[b] *Department of Industrial Engineering, Arizona State University, Tempe, AZ 85287-5906, USA*

Received 21 December 2004; accepted 14 June 2005
Available online 4 October 2005

Abstract

The dynamics and throughput of a bucket brigade production system is studied when workers' speeds increase due to learning. It is shown that, if the rules of the bucket brigade system allow a re-ordering of its workers then the bucket brigade production system is very robust and will typically rebalance to a self-organizing optimal production arrangement. As workers learn only those parts of the production line that they work on, the stationary velocity distribution for the workers of a stable bucket brigade is non-uniform over the production line. Hence, depending on the initial placement of the workers, there are many different stationary velocity distributions. It is shown that all the stationary distributions lead to the same throughput.

Keywords: Bucket brigades; Dynamics of production systems; Dynamical analysis

1. Introduction

A bucket brigade is a linear production line in which each worker picks up a job and processes it at each station until he gets "bumped" by a downstream worker. Whenever the last worker in the line finishes a job, he initiates a reset of the production line: Each worker takes over his predecessor's job and the first worker starts a new job. Such bucket brigade production systems have been successfully used in many different manufacturing environments ranging from apparel manufacturing to warehouse order picking operations. Bartholdi and Eisenstein (1996) coined the term "bucket brigades" and provided the first comprehensive analysis of the dynamics of such systems. The authors showed that if workers can be sequenced from slowest to fastest so that each worker is strictly faster than his predecessor at every point along the production line, then the bucket brigade is self-balancing (i.e., a stable partition of work among

* Corresponding author. Tel.: +1 480 965 2906; fax: +1 480 965 8692.
E-mail address: esma.gel@asu.edu (E.S. Gel).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2005.06.052

workers will emerge such that each worker repeatedly executes the same interval of work content). Furthermore, the bucket brigade yields optimal throughput.

Since Bartholdi and Eisenstein (1996), the dynamics and performance of several other bucket brigade environments have been studied. Bartholdi et al. (1999) describes all possible asymptotic behavior of two- and three-worker bucket brigade production lines as a function of the workers' relative speeds. Bartholdi et al. (2001) addresses the case of stochastic processing times, and proves a similarity between the deterministic and stochastic systems as the number of stations goes to infinity. Armbruster and Gel (2004) considers production environments in which workers cannot be ordered slowest to fastest since their speeds (and hence the relative speeds of the workers) change from one region of the production line to another. The study presents conditions under which bucket brigades continue to be effective. Bartholdi and Eisenstein (2004) discusses the importance of walk-back times on the convergence properties of a bucket brigade and shows that in the existence of walk-back times, a bucket brigade line is capable of complex dynamics.

One of the basic assumptions of almost all previous work on bucket brigades is that the worker speeds are constant over time (though possibly a stationary stochastic process). However, almost all production systems involve learning by workers, and as a result, the speed of processing typically improves over time, as more and more units are processed. Some examples are systems with complex tasks requiring continuous reinforcement of skills, systems with high labor turnover and absenteeism typical for instance in the industries along the US–Mexican border (the Maquiladoras discussed in Hutchinson et al. (1997)), and systems with very low volume or high customization.

Regardless of the reason, worker speeds change over time in many production environments where bucket brigade mode of production can be a viable alternative. In this study, we consider the case of improving worker speeds due to learning. While there has been some consideration of learning and the use of bucket brigades in such environments (Munoz and Villalobos, 2002; Villalobos et al., 2003), prior work on bucket brigades does not provide a comprehensive analysis of the dynamics of a bucket brigade operating under such conditions. Our objective is to study the dynamics of bucket brigades with learning and provide some insights on the management of such systems. We maintain most of the basic assumptions from Bartholdi and Eisenstein (1996): we consider a production line with fully cross-trained workers and continuous tasks. Handovers from one worker to the next are possible at any position along the line and workers walk back with infinite speed to take over jobs. All processing times are deterministic. However, variations in processing times still exist due to different worker velocities.

In addition to these basic assumptions, we consider the following scenario to determine the effect that worker learning has on the dynamics and stability of a bucket brigade. We assume that N workers are ordered from slowest to fastest with constant speeds $v_1 < v_2 < \cdots < v_N$ along the production line. Hence, we assume an optimally working bucket brigade arrangement. To this bucket brigade, an additional worker with no prior training is added. We will call this worker the *new worker* and refer to this person as "he." We assume that the new worker is initially completely untrained, and hence is the slowest worker in the group, i.e., $v_{\text{new}}(0) = v_\ell < v_1$. However, we also assume that the new worker may, over time, learn and improve his speed in the regions he works on. We assume that the work along the production line is sufficiently different that a worker only learns the part of the production line that he actually performs. We are interested in how this learning and improvement in the new worker's speed affects the dynamics of the bucket brigade production line.

There is vast amount of research studying the organizational and individual learning in manufacturing settings. Individual based approaches have focused on understanding the mechanisms by which learning occurs at the individual level, and provided important insights on the appropriate mathematical models of individual learning (Nembhard and Uzumeri, 2000). In our study, we considered the use of various forms of learning models, and primarily used an exponential model

(Mazur and Hastie, 1978) to represent the improved speed due to learning. The velocity of the learning worker at time t is expressed as

$$v(t) = v_\ell + (v_h - v_\ell)(1 - e^{-t/\tau}), \tag{1}$$

where v_h denotes an inherent maximal speed for the new worker. The parameter τ is a measure of the ease of learning: a large value of τ reflects a difficult task that needs lots of reinforcement, whereas a small value of τ implies an easy task that can be quickly learned. The exponential model has the advantage that an initial velocity and a final velocity which the worker reaches after enough training are well defined.

We have also studied the log-linear model (Wright, 1936; Yelle, 1979), which describes the improvement of the worker speed as a function of the number of times, k that a worker has performed a particular task as

$$v(k) = v_\ell k^m, \tag{2}$$

where m is commonly referred to as the "learning index," with $0 < m < 1$. In contrast to the exponential model, the log-linear model allows unlimited increase of worker speed over time. However, we have found that due to simulation restrictions or due to measurement errors in reality, we effectively reach a limiting velocity, v_h. In addition, our results with both learning models were quite similar; in other words, the dynamical behavior were almost the same for the log-linear model and the exponential model. Hence, for the sake of brevity, the results from models with the exponential learning curve model are presented in this manuscript.

Section 2 considers the case of inserting a new worker to an established bucket brigade line. In Section 2.1, we discuss typical simulations for the addition of a new worker to an established bucket brigade with passing. In Section 2.2, we present theoretical results for the throughput performance as well as for the self-organization properties in such a case. Section 2.3 discusses the addition of a new worker without the possibility of switching or reorganization of the bucket brigade. We refer to this case as *blocking*. As expected, the *correct* initial placement of the new worker is shown to be crucial. In Section 3, we return to the passing/switching case and briefly discuss the situation where all workers in the bucket brigade are new and are learning to improve their processing speeds. We conclude with several managerial insights that provide guidelines on the operation of such bucket brigades.

2. Addition of a single new worker

Adding a new worker to the bucket brigade at any location other than the first will disturb the *slowest to fastest* ordering of the workers. At the time of insertion of the new worker, he is the slowest (wherever he is placed) among the workers. Hence, according to the rule of ordering workers slowest to fastest, he should clearly be placed at the starting position. On the other hand, if v_h is large enough, at some time the new worker will have acquired additional skills such that he should not necessarily be working at the beginning of the line any more (since he may be faster than the worker that was in the starting position originally).

In order to allow the bucket brigade to self-organize itself beyond just the distribution of work, we append the rules of a basic bucket brigade and allow workers to pass each other. This protocol is referred to as *switching* or *passing* in Armbruster and Gel (2004) and has been shown to enable self-organization in cases in which self-organization would not have been possible otherwise. Under the traditional bucket brigade rules, if a worker catches up with a downstream worker he gets blocked. Hence, the order of the workers are preserved at all times. When passing is allowed, a worker, if he catches up with a downstream worker, can pass the downstream worker and continue processing his job down the line, rather than being blocked. As a result, the position of a worker may be dynamically adjusted, and he will not a priori have a specific position number in the line.

The most obvious way to implement such a rule is to require that any worker that gets blocked by a slower worker downstream exchanges position with that worker. We assume that such an exchange is instantaneous and does not lead to lost production.

2.1. Bucket brigades with passing: Simulations

Here we present simulation results for the addition of a single new worker to an existing bucket brigade to illustrate the typical behavior and to build intuition on how the bucket brigade behaves. We have simulated both learning models (log-linear and exponential) and coded them in Matlab. The detailed handover protocol is the following: Workers can pass each other freely without any delay. Whenever a worker reaches the end of the line everybody just takes the workpiece from the worker immediately preceding him and the first worker in the line starts a new item. In that way the worker order is not fixed and re-adjusts itself.

We explored a large range of values of the various parameters such as the worker speeds, initial positions, and learning rates. One such instance is a bucket brigade with a new worker with a starting speed of $v_\ell = 1$ and a maximal speed of $v_h = 1.85$ on all of the production line and four existing workers with speeds $(v_1, v_2, v_3, v_4) = (1.4, 1.7, 2.0, 2.3)$, ordered from slowest to fastest. Fig. 1 presents the velocity distribution when it does not seem to change any more (typically after about 50 to a couple of hundred resets of the bucket brigade) for the exponential learning curve model with $\tau = 0.7$ and the new worker starting and finishing at position 2.

As can be seen from the figure, the new worker reaches his maximal velocity for a certain interval along the production line. Clearly, the worker is not working outside that interval when the bucket brigade reaches steady state. The additional steps in the velocity distribution reflect transient learning in a part of the production line that is not covered in steady state any more. The bucket brigade rearranges itself to a new fixed point that includes the two handover points generated by the new worker. The handover points have to be at the boundary of the interval of maximal velocity or inside it.

An interesting result of these simulations is that for large sets of initial conditions the bucket brigade maintained its initial order and the new worker generated an interval of competency consistent with his initial placement. In particular, the stable final order does not necessarily reflect the usual slowest to fastest arrangement. This is also shown in Fig. 1 where the second worker is faster than the third worker. For the specific speed

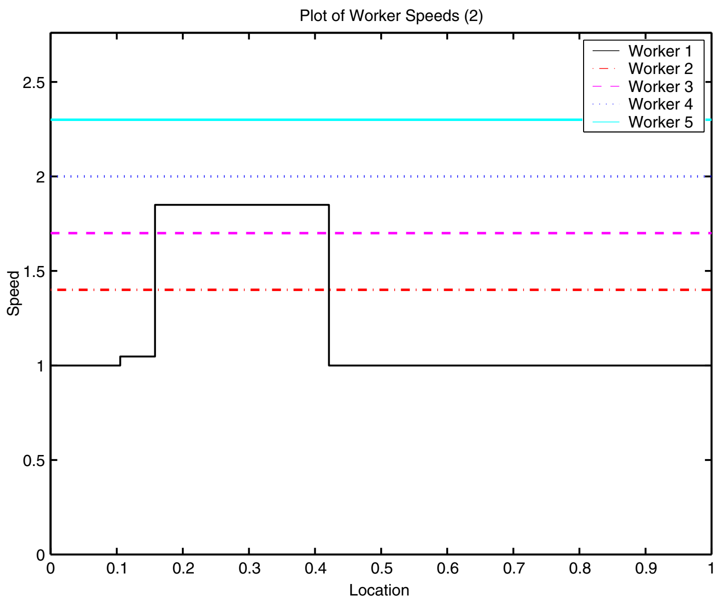

Fig. 1. Steady state worker speeds for placing the new worker at position 2.

distribution chosen above, only when we placed the new worker at the very end of the line could we not find initial conditions that stabilized him there. He typically switched position and became the worker at position 4. We will come back to these results in a remark following our theoretical discussion in the next section.

When we strongly increase the final velocity of the new worker such that he will become by far the fastest worker then the incidences of switching occur more and more often during the transient learning phase and the eventual organization of the bucket brigade will be the one from slowest to fastest.

Our simulations also reveal that the learning rule does not play a role for the eventual form of the stable bucket brigade. Simulations indicate that it also rarely changes the type of transient behavior. We could not find any significant differences for the log-linear learning model and the exponential model. Similarly, we find that the learning speed τ or index m has an influence on the transient but not on the steady state behavior.

2.2. Theoretical results

The simulations in the last section illustrate the following theorem on the self-organizing behavior and throughput of the bucket brigade.

Theorem 1. *Consider the exponential learning model defined in Eq. (1). As $t \to \infty$, the bucket brigade with the above switching rule has only fixed points as possible attractors. All such fixed points have the same throughput, independent of the initial worker ordering.*

Proof. Let us consider n workers. The dynamics of the motion of the workers can best be understood through the reset map: Every time the last worker finishes product number r, the position $x_1(r), \ldots, x_{n-1}(r)$ of all the $n-1$ other workers is registered in a vector $\mathbf{x}(r) = (x_1(r), \ldots, x_{n-1}(r))$. Since all the velocities are deterministic, we can, in principle, calculate the positions of all workers when the next product is finished from the positions at the current reset, i.e., $\mathbf{x}(r+1) = \mathbf{f}(\mathbf{x}(r))$. The map $\mathbf{f}(\mathbf{x})$ is called the reset map, and a fixed point $\mathbf{x}^*$ of this reset map corresponds to a balanced bucket brigade where all workers cover the same part of the production line for every iteration.

The theorem can be proved by contradiction. Let us assume that the dynamics of the new worker as $t \to \infty$ does not correspond to a fixed point but to a periodic orbit or a chaotic orbit. In that case, there is a maximal and minimal value on the production line, $x_{\min}$ and $x_{\max}$, such that the orbit stays inside the interval $I_{\max} = [x_{\min}, x_{\max}]$. However, in the limit as $t \to \infty$, this interval will have been covered infinitely often, and hence inside that interval the new worker will have attained its maximal speed, v_h. Therefore, the dynamics for the new worker is completely linear. Since all the other workers also have constant speeds the motion of all workers is completely linear. Hence the resulting reset maps will be completely linear and therefore we will not have a periodic or chaotic orbit. Hence, the dynamics of the new worker in the limit is characterized by a fixed point of the reset map. The worker will completely stay inside the interval $I_{\max}$ at each iteration with $x_{\min} \leqslant x^* < y^* \leqslant x_{\max}$ and x^*, y^* being the start and finish points for the new worker, i.e., the two of the coordinates of the fixed point.

To prove the second part of the theorem, we consider a balanced bucket brigade, balanced on an arbitrary fixed point. The fact that the bucket brigade is balanced implies that the throughput for all the workers upstream from the new worker (which is equal to $\Sigma_i v_i / x^*$), the throughput of the new worker (which is equal to $v_h/(x^* - y^*)$), and the throughput of all the workers downstream from the new worker (which is equal to $\Sigma_j v_j / (1 - y^*)$) are equal. That is,

$$\frac{\Sigma_i v_i}{x^*} = \frac{v_h}{y^* - x^*} = \frac{\Sigma_j v_j}{1 - y^*}. \tag{3}$$

Eq. (3) can be solved to determine x^* and y^*, leading to an overall throughput of

$$\mathrm{TP} = \Sigma_i v_i + \Sigma_j v_j + v_h, \tag{4}$$

which is independent of the position of the new worker. □

While Theorem 1 is concerned with the throughput, a self-organizing bucket brigade also requires that those fixed points are attractors. From a practical point of view, the fixed points should be global attractors (or at least have a large basin of attraction) so that the need for management intervention is minimized. Hence, a necessary condition for practical usefulness of a bucket brigade is its "local stability," i.e., existence of an open neighborhood in phase space that gets asymptotically attracted to the fixed point.

The fixed point discussed in Theorem 1 is characterized by the discontinuous velocity distributions as shown in Fig. 1. Therefore, in order to study its stability we consider a velocity distribution for the new worker that is of the form

$$v_{\text{new}} = \begin{cases} v_\ell & 0 < x < X \\ v_h & X < x < Y \\ v_\ell & Y < x < 1 \end{cases} \tag{5}$$

and assume that there exists a fixed point inside the interval $[X, Y]$. In order to be able to proceed analytically and generate some qualitative understanding, we approximate the interaction of the new worker with the rest of the bucket brigade in the following way: We replace the p workers upstream from the new worker and the s workers downstream from the new worker by one worker, each of whom has a mean velocity of

$$\bar{v}_1 = \frac{\Sigma_{i=1}^{p} v_i}{p} \quad \text{and} \quad \bar{v}_2 = \frac{\Sigma_{j=1}^{s} v_j}{s}, \tag{6}$$

respectively.

The resulting three-worker system is characterized by the three positions $(x_1, x_{\text{new}}, x_2)$. It has a fixed point P^* at $(x_1^*, x_{\text{new}}^*, x_2^*)$. The map relating subsequent reset points now becomes a 2-d map, $F : (0, x_{\text{new}}(t_n), x_2(t_n)) \to (0, x_{\text{new}}(t_{n+1}), x_2(t_{n+1}))$. If $(x_{\text{new}}^*, x_2^*) = (X, Y)$ then F is piecewise linear with four different regions all meeting at the fixed point. As a result, F is not differentiable at P^* and the stability of P^* is very hard to determine. This difficulty is really an artifact of our assumptions: We are assuming that a worker learns only on the part of the production line that he has actually worked on. In that way, the sharp discontinuity in speeds (skills) may coincide with the handover points.

However, we can assume that the actual handover points are inside the interval $[X, Y]$ for two reasons: (i) It is highly likely that during the learning phase the new worker has worked on a significantly larger portion of the production line at some time than in the final equilibrium state, (ii) we can assume that the tasks on the production line have some similarity. Hence, if we discretize the production line into finite production units, and assume that a worker has learned to do the whole unit once he has worked on part of a unit, we will have the discontinuity in speeds at an end of a production unit at X and Y, which typically will not coincide with the handover points x_{new}^* and x_2^*. In particular,

$$X < x_{\text{new}}^* < x_2^* < Y. \tag{7}$$

Hence, we can perform the local stability analysis for a system of three workers, each of them having a uniform velocity of $\bar{v}_1$, v_h and $\bar{v}_2$, respectively. It has been shown by Bartholdi et al. (1999) that the fixed point is stable if and only if

$$\begin{aligned} \bar{v}_2 &> \bar{v}_1, \\ \bar{v}_2 &> v_h - \bar{v}_1. \end{aligned} \tag{8}$$

Fig. 2 shows the stable regime in $(\bar{v}_1, \bar{v}_2)$ parameter space for a fixed $v_h = 1$. We find that even when v_h is larger than $\bar{v}_2$, the fixed point may still be stable, as the above inequalities imply. Outside

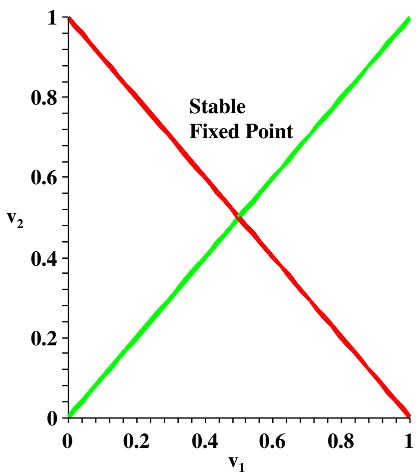

Fig. 2. The stability conditions given Eq. (8), plotted for $v_h = 1$. The fixed point is stable for $(\bar{v}_1, \bar{v}_2)$ above both lines.

the stable regime, the fixed point will be unstable which eventually will lead to a re-ordering of the bucket brigade.

Remarks.

- It is instructive to check the conditions in Eq. (8) for the velocity distributions of Fig. 1. The situations where the new worker is placed in position 2, 3 or 4 are consistent with stability as given by Eq. (8). In the cases where the new worker is either first or last, the limiting velocity $v_h = 1.85$ is equal to the average velocity of all the other workers $\bar{v} = (1.4 + 1.7 + 2 + 2.3)/4 = 1.85$. Since we start with a low velocity $v_l = 1$ the transient stabilizes the position of the new worker at the start of the bucket brigade while it destabilizes his position at the end of the bucket brigade, which leads to the interchange of the position of the new worker with the worker at position 4.
- Even when the fixed point is locally stable, it may not be globally stable. In particular, assume that $v_l = \epsilon$, i.e., very small. Hence, any perturbation that pushes the new worker outside the interval $[X, Y]$ will have him crawl along relative to the other workers. This implies that the upstream worker will catch up with him and will hence "interchange" positions with him. Therefore, the perturbations that keep the fixed point stable are restricted to the interval $[X, Y]$.

Placing a worker at a position that is unstable as he learns may lead to quite interesting and complicated dynamics. Fig. 3 shows the transition of a new worker at position 3 to position 4. Plotted are the handover points as a function of time. We see transient periodic orbits that seem to become unstable in a period doubling cascade known from chaotic maps. The convergence point of all the period doublings is the actual transition point changing the order of the bucket brigade.

There are several important observations that can be made from the simulations depicted in Fig. 3:

- For a new worker placed in a position that will, after learning, lead to a stable bucket brigade, there are typically very few transient situations in which the bucket brigade re-orders itself.
- For a new worker placed into a position that will, after learning, not lead to a stable bucket brigade, the bucket brigade typically continues to re-order until it has a final order from slowest to fastest.
- While the bucket brigade generates a stable equilibrium as time increases, the transient might be quite long and will have suboptimal production.

The last point implies that the dynamics of the bucket brigade may call for timely management intervention. In many cases such an intervention will be quite obvious. Looking at Fig. 3, we can already discern at $t = 50$ that the limiting velocity for the new worker is the highest among all workers. Hence, a manager might want to reorder the bucket brigade at that point rather than waiting for it to self-organize.

2.3. Bucket brigades with blocking

When workers are not allowed to pass each other, and hence no re-ordering is possible, the system becomes very dependent on the initial conditions. In particular, we have found that for any initial position of the new worker, if his final velocity at that position will generate a stable bucket brigade, then the bucket brigade will self-organize to that fixed point. If, however, the worker is placed in a position that becomes unstable as he learns, some workers in the bucket brigade will become blocked. In that case, the dynamics of the bucket brigade will become periodic or chaotic, and the throughput will become suboptimal.

Fig. 4 shows the handover points for a bucket brigade with 5 workers and blocking. The new worker is in position 2 and is, after a short while, the fastest worker, which is indicated by the fact that the interval between the handover point 1 and 2 is the largest among all production intervals. While the transient looks quite stable, after about 140 iterations the fast worker at position 2 catches up with the worker at position 3. After that time, the handover points lose all regularity and appear to follow a chaotic pattern.

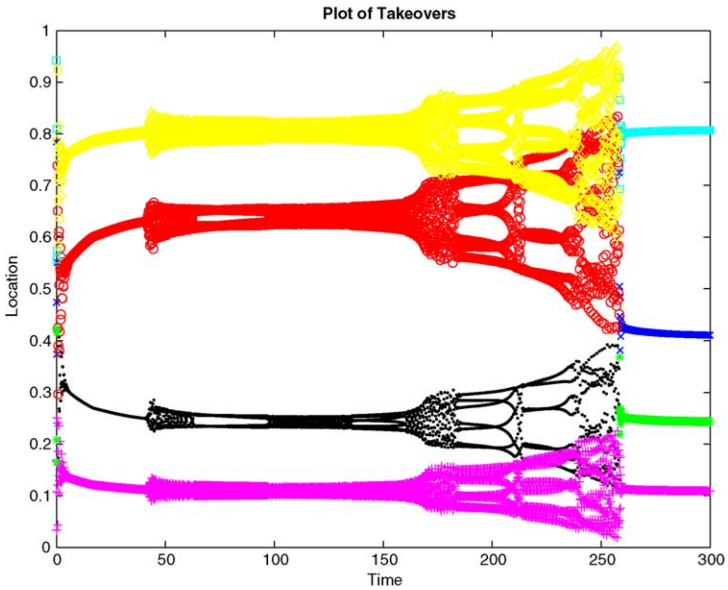

Fig. 3. Handover positions for five workers: a new worker at position three switches to position 4 via a period doubling transient.

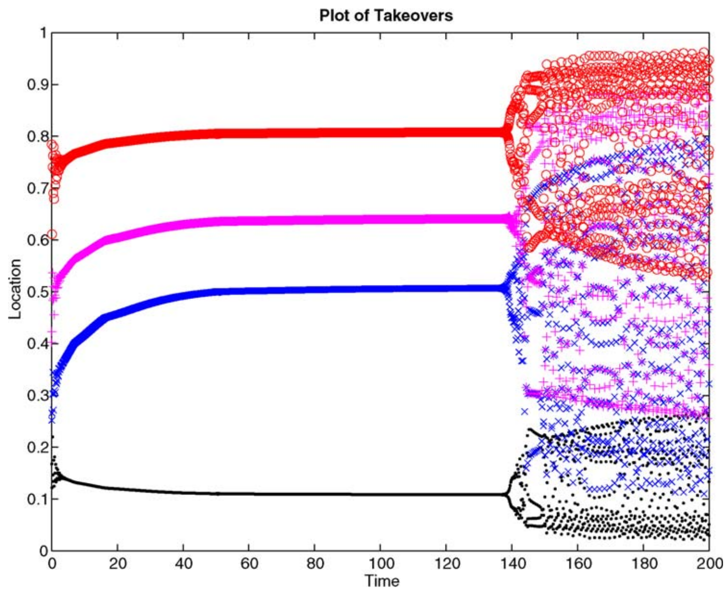

Fig. 4. A worker at the wrong position with blocking leading to a chaotic dynamics.

3. Learning by all workers with passing rule

Theorem 1 can easily be generalized to cover a bucket brigade with all "new workers" who are simultaneously learning and improving their processing speeds. We again assume that upon blocking each other, workers switch position, i.e., pass each other.

Corollary 1. *Consider a bucket brigade where all members are learning. If passing is allowed, then as* $t \to \infty$ *the bucket brigade has only fixed points as possible attractors. All such fixed points have the same throughput, independent of initial worker ordering.*

Proof. The basic proof idea of Theorem 1 still holds. Asymptotically, each worker will have his maximal (and constant) speed on a section of the production line that contains the interval of production that he is repeating in the steady state. Hence, the reset map will asymptotically evolve to a linear map containing only a stable fixed point. Since none of the fixed points allow blocking (otherwise the order would be switched) they all must have the same throughput. □

Fig. 5(a) and (b) show an illustration of such a learning bucket brigade. We started five workers at arbitrary locations all with an initial speed of 1. The limiting or maximal speeds for workers 1 through 5 are $(2, 3, 4, 5, 6)$, respectively. The resulting stable organization is given by worker 1, followed by 2, 4, 3 and 5. Fig. 5(a) shows the resulting velocity profile and Fig. 5(b) shows the time evolution of the handover points.

4. Insights and conclusions

We have studied the dynamics of bucket brigade production lines when worker speeds improve over time due to learning. We first considered the case of adding a single inexperienced worker to an otherwise stable bucket brigade. The most important conclusion from our study is that the self-organizing principle of the bucket brigade is very robust: The bucket brigade self-organizes not only to an optimally split workload assignment for all workers, but it can also self-organize to the optimally stable positions of the workers through a switching rule that allows the workers to trade places. With such a rule, the initial positioning of the new worker is irrelevant, which implies that significant managerial intervention is not necessary for effective operation of the line.

In an environment where passing is not possible and blocking may occur, a stable bucket brigade typically emerges if the final velocity of the new worker is not too different from his neighboring workers. In cases where this does not hold, some management intervention in the form of careful ordering of workers is required for optimal (or effective) operation of the line.

We have also extended our results to the case in which all workers are learning at the same time, in a brand new bucket brigade line. We find that a bucket brigade where all workers are learning will always lead to self-organized production, which is a strong and useful result that supports the use of bucket brigades in such environments. Several interesting management insights follow from our analysis:

- We can use the arguments leading to Eq. (8) to estimate the best position to enter a new worker into a bucket brigade: Determine a reasonable estimate of the final velocity of the new worker. The new worker will split the bucket brigade into an upstream and a downstream part. Determine the mean velocity for the workers in both parts and place the new worker such that Eq. (8) is satisfied.
- The previous insight is especially important for bucket brigades that do not allow switching.
- Although the bucket brigade will achieve steady state from all initial positions, the speed of convergence is a major issue. During the transient phase the bucket brigade may not have an optimal throughput. Hence, a fast transient has the potential to significantly improve the performance of the production line. This implies that tasks that are easy to learn are probably more suited for a bucket brigade organization than hard and complicated tasks that lead to prolonged transients.
- During the transient phase switching happens in two ways: A new worker that starts slow at a downstream position will get caught by the upstream worker and they will interchange positions. This will happen during the very first few iterations when the worker is still significantly slower than the others. Afterwards, the worker will learn and improve his speed. If

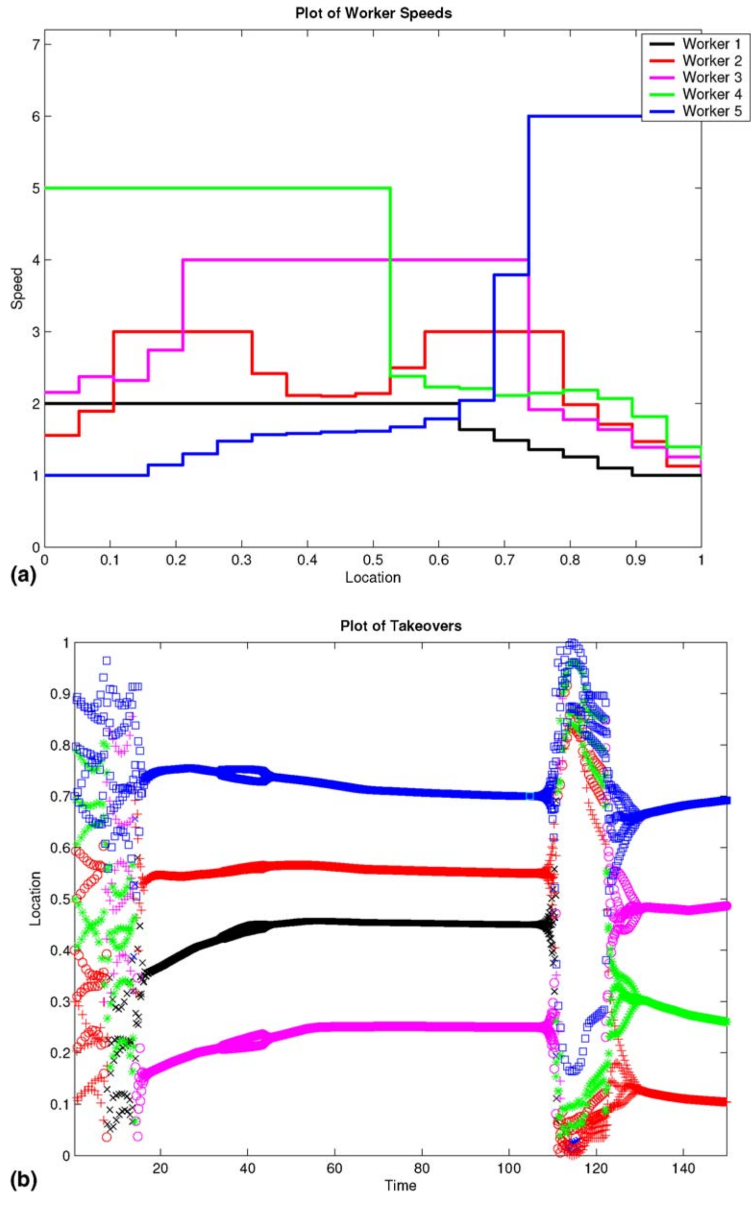

Fig. 5. (a) Steady state worker speeds for a five-worker bucket brigade in which all workers are learning simultaneously. (b) Time evolution of the handover points for the same bucket brigade.

the speed distribution is such that the worker will catch up with the next downstream worker they will interchange position. However, that transient process happens on a much slower time scale, since the increase of the speed will only happen on the leading edge of the current competency of the new worker. Fig. 3 illustrates these two transient processes nicely: The initial transient is over within at most 10 iterations after which it will take the new worker 250 iterations to trade places with the next downstream worker. In such a case, judicious intervention by a manager, moving the new worker to a more suitable place will cut the transient time significantly. Once a good estimate of the final worker speed can be made, a temporary fixing of the worker orders, inhibiting place switching, may give the new worker enough time to learn in his eventual production area and lead to a fast stabilization of the bucket brigade.

Our analysis characterizes the dynamics of bucket brigades with learning and provides several important operational guidelines on the design and operation of such systems. While we have not presented results for an arbitrary number of new workers, we expect that similar insights follow for those cases. One area of future work is the consideration of forgetting, in addition to learning. (Forgetting alone would result in a similar dynamics to the one we presented in this manuscript.) We believe the combined effect of learning and forgetting is likely to result in complex dynamics, which needs to be better understood since the behavior may limit the usefulness or practicality of bucket brigade mode of production.

Acknowledgement

This work was partially supported by a grant from the National Science Foundation (DMS-0204543). We thank John J. Bartholdi, III for helpful discussions and feedback.

References

Armbruster, D., Gel, E.S., 2004. Bucket brigades revisited: Are they always effective? European Journal of Operational Research, in press, doi:10.1016/j.ejor.2004.08.044.

Bartholdi, J.J., Eisenstein, D.D., 1996. A production line that balances itself. Operations Research 44 (1), 21–34.

Bartholdi, J.J., Eisenstein, D.D., 2004. Chaos and convergence in bucket brigades with finite walk-back velocities. Technical report, Dept. of Industrial Engineering, Georgia Institute of Technology, Atlanta, GA 30332.

Bartholdi, J.J., Bunimovich, L.A., Eisenstein, D.D., 1999. Dynamics of two- and three-worker "bucket brigade" production lines. Operations Research 47 (3), 488–491.

Bartholdi, J.J., Eisenstein, D.D., Foley, R.D., 2001. Performance of bucket brigades when work is stochastic. Operations Research 49 (5).

Hutchinson, L.F., Villalobos, J.R., Bernvides, M.G., 1997. Effects of high labor turnover in a serial assembly environment. International Journal of Production Research 35 (1), 3201–3223.

Mazur, J.E., Hastie, R., 1978. Learning as accumulation: A reexamination of the learning curve. Psychology Bulletin 85 (6), 1256–1274.

Munoz, L., Villalobos, J.R., 2002. Work allocation strategies for serial assembly lines under-high labor turnover. International Journal of Production Research 40 (8), 1835–1852.

Nembhard, D.A., Uzumeri, M.V., 2000. Experiential learning and forgetting for manual and cognitive tasks. International Journal of Industrial Ergonomics 25, 315–326.

Villalobos, J.R., Mar, L.R., Gutierrez, M.A., Montano, A., 2003. The use of dynamic work sharing production methods to reduce the impact of labor turnover in serial assembly lines. Technical report, Dept. of Industrial Engineering, Arizona State University, Tempe, AZ 85287.

Wright, T.P., 1936. Factors affecting the cost of airplanes. Journal of Aeronautical Sciences 3 (4), 122–128.

Yelle, L.E., 1979. The learning curve: Historical review and comprehensive survey. Decision Sciences 10, 302–328.