



Production, Manufacturing and Logistics

# Bucket brigades revisited: Are they always effective?

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## Abstract

Previous work on the dynamics of bucket brigades has focused on systems in which workers can be ordered with respect to their speeds and where this ordering does not change throughout the line. While this assumption is valid in most environments, it may not be satisfied in some. We consider such environments and explore the conditions under which bucket brigades continue to be effective (compared to a traditional static allocation) with respect to self-balancing behavior and throughput performance. A two worker bucket brigade is studied where one worker is faster than the other over some part of the production line and slower over another part of the line. We analyze the dynamics and throughput of the bucket brigade in two environments with passing and blocking. We present the dynamics of the system in each region of the parameter space and provide insights and operating principles for the implementation and management of the bucket brigades under various scenarios.

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## 1. Introduction

The widespread use of cross-training in industrial settings has led to a growing interest to study operational control in systems with multi-skilled, or flexible workers. In particular, control policies that are defined by a set of rules that tells each worker what to do “next” have received

significant attention by industry due to their ease of implementation and management. The Toyota Sewn Products Management System (TSS) is one of the most widely studied architectures of this kind. TSS is employed regularly by manufacturers of sewn products in modules that are used in the finishing and assembly of cut parts into a sub-assembly or finished garment. The production modules are typically U-shaped, and workers process garments as a team. In a TSS line, each worker picks up a task and processes (also carries) it at each station until he gets bumped by a

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downstream worker. There are no additional WIP buffers kept.

The number of machines in a TSS line typically ranges from 2 to 16 with an average of 2.5 machines per worker (Bischak, 1996). In addition to apparel and garment industry, TSS lines have also been shown to perform robustly in certain types of warehousing environments (Bartholdi and Eisenstein, 1996).

The term “bucket brigade” was coined by Bartholdi and Eisenstein (1996) for TSS lines in which the workers are sequenced from slowest to fastest. The authors provided the first comprehensive analysis of the dynamics of such systems and showed that the bucket brigade is self-balancing; that is, eventually a stable partition of work among workers will emerge such that each worker repeatedly executes the same interval of work content.

The main result in Bartholdi and Eisenstein (1996) states that if workers can be sequenced from slowest to fastest so that each worker is strictly faster than his predecessor at every point along the production line, then there is a stable fixed point that the system will converge to, independent of the initial positions of the workers. Bartholdi et al. (2001) addressed the case of stochastic processing times, and proved a similarity between the deterministic and stochastic systems as the number of stations goes to infinity. Bartholdi et al. (1999) focused on two- and three-worker bucket brigade production lines and described all possible asymptotic behavior as a function of the workers’ relative speeds, which is assumed to be constant over the entire line. For two workers, the authors showed that only two modes of asymptotic behavior exist: a fixed point with optimal production rate or a period-two orbit with suboptimal production rate. For three workers, the dynamics can be quite varied with cycles ranging from 1 (i.e., self-balancing) to 30,000, depending on the relative speeds of the three workers.

The most well-known work on bucket brigades, Bartholdi and Eisenstein (1996), has shown that bucket brigades are not only self-balancing, but also results in optimal system performance (i.e., throughput). One important thing to note, however, is that the current state-of-the-art in the liter-

ature addresses systems in which the worker speeds are “uniformly dominated”. In other words, it is possible to order workers from slowest to fastest, since the ordering of the worker speeds do not change from one region of the production line to another. However, it is an important practical consideration to address whether bucket brigades perform effectively for systems in which this assumption is not justified.

In fact, there are several real life systems in which workers have different speeds in different portions of the line. For example, systems with high labor turnover frequently encounter situations where newly hired workers have skills that are applicable only to parts of the production line (Hutchinson et al., 1997). Another example are systems with stations that require extensive specialization or training. Since the bucket brigade assumes that all workers, if necessary, work at all stations, having one such station in the middle of the line, for example, can create some implementation problems. One such example was given by Bartholdi (private communication); this was an apparel manufacturing plant with a specialized station in the middle of the line, at which only one worker was sufficiently specialized to perform the operations in a fast enough pace. In such a case, the practical questions are whether one should employ a single bucket brigade, two separate bucket brigades with a stationed worker in the middle, or whether bucket brigade policy is a good idea or not in the first place.

In this paper, we study systems in which a worker’s speed at a particular task significantly depends on the type of the task, and hence the worker’s speed relative to another worker can change drastically from one portion of the line to the next. That is, a worker maybe faster on one portion of the line and significantly slower on another. It is important to note that the case we study, in addition to the environments discussed above, will probably occur naturally after some time in any bucket brigade line. If the workers are organized so that the line is self-balancing (slowest to fastest, according to Bartholdi and Eisenstein, 1996), each worker will perform most or all of his/her work in the same portion of the line, where he/she will become more proficient, and probably, faster. The

slowest to fastest ordering may still be preserved (if all workers learn at the same pace), but any changes to the line, such as adding or removing workers, may cause the relative velocities to vary by position on the line.

Our objective is to determine whether these systems exhibit the same type of dynamical behavior observed in previous work, assess the performance of bucket brigades under such conditions, and provide managerial insights and principles for the implementation of bucket brigades in such environments. In particular, we consider a two-worker bucket brigade where each worker has his/her own regions of specialization, defined by a breakpoint in the line. We assume that a worker is faster than the other worker in his/her own region of specialization.

We consider two fundamentally different types of operating rules to represent different types of production environments. The first one of these environments allows passing or overtaking of the downstream worker by an upstream worker. Warehouse order picking seems to be a prototypical case for this assumption. To our knowledge, this represents the first analysis of such a bucket brigade environment, although we are aware of some more recent work by [Bartholdi and Eisenstein \(2002\)](#), in which the authors assume that workers can overtake one another since each worker has his/her own workstation. One criticism of the passing assumption may be that it requires duplicate tooling (and hence its applicability may be limited in real-life systems). While this may be true in some cases, in some others where there does not exist duplicate tooling, it may be possible for the workers just to exchange workpieces to enable passing. Furthermore, we show that enabling passing may convert an unstable bucket brigade into one that self-balances, or “self-organizes”. Hence, in situations where *global* self-organizing behavior is important, additional equipment costs, if necessary, may be justified. However, we also show that if the management is able to collect rough-cut information on workers’ speed structure, it is possible to avoid costs of duplicate machinery: one can simply start the bucket brigade “close enough”, and that will be enough to enable self-organizing behavior. Note that we use the term

“self-organizing” to include and refer to a more general set of dynamical behavior, such as a period-two orbit, than the term self-balance implies.

Following the passing environment, we provide the analysis of bucket brigade environments in which passing is not allowed; that is, the order of workers has to be preserved at all times. This means that under the worker speed structure we are considering, blocking of workers can happen, since the upstream worker can “catch-up” with the downstream worker.

The simplistic two-worker system enables us to analytically evaluate the performance of the bucket brigade system with respect to two criteria: self-balancing (or self-organizing) behavior and throughput. Based on this analysis, we reach important practical considerations, which have not been discussed in previous literature. We note, however, that our analysis is useful to many real life systems since many bucket brigade systems consist of three or two workers ([Bartholdi et al., 1999](#)).

The next section outlines our assumptions and provides a formalization of our model of worker speeds. In Section 3 we provide analysis of the bucket brigade with passing and present general results on the behavior. Section 4 includes similar results for the dynamics of bucket brigades under the assumption of blocking. In Section 5 we provide several useful managerial insights for the implementation and operation of bucket brigades. Finally, in Section 6 we provide some concluding remarks and insights, and suggest some areas for future work.

## 2. Modeling assumptions

Our basic model and assumptions are similar to those made by [Bartholdi and Eisenstein \(1996\)](#). We consider a production line with fully cross-trained workers and continuous tasks. The processing times are assumed to be deterministic and each worker has a velocity function that gives his/her instantaneous velocity at each point along the production line. Workers walk back to take over jobs with infinite velocity (zero walk-back times). However, our work differs from that of

Bartholdi and Eisenstein (1996) in the sense that we consider systems in which the velocity of one worker does not uniformly dominate that of another at every point along the production line. Note that this implies that ordering workers slowest to fastest is not possible, since the ordering changes from one region of the production line to another.

Specifically, we consider a bucket brigade with two workers, which we denote as worker A and worker B. For convenience, we refer to worker A as a male and worker B as a female. We scale work content and time such that worker B has a uniform speed along the production line which we set to 1. This is equivalent to saying that only the relative speed of worker A to worker B matters. We call B the reference worker and assume that she always starts the line. Work content of a job (i.e., the amount of work already performed on the job) is denoted by  $x$  where  $x = 0$  reflects a new entry into the production line and  $x = 1$  corresponds to a finished product. We assume that the speed of worker A is a function of his location on the line in such a way that he has speed  $v_1(x)$  on the interval  $[0, X]$  and  $v_2(x)$  on the interval  $[X, 1]$ . Furthermore  $v_1(x) > 1$  and  $v_2(x) < 1$  or  $v_1(x) < 1$  and  $v_2(x) > 1$ . Since we can always rename workers A and B, we therefore cover all possible arrangements of the bucket brigade with respect to the workers' speeds.

We are interested in two major issues: (i) which arrangement or sequencing of the workers will lead to a self organizing production line, (i.e., a line that is stationary in some sense from almost all initial conditions), and (ii) which arrangement leads to the best average throughput. Throughout the paper, we will assume that work is continuous in a similar spirit to Bartholdi and Eisenstein (1996), however, we note that the analysis under the discrete work or discrete workstations case yields similar results, details of which are omitted.

### 3. Dynamics for the passing case

In this section, we extend the term bucket brigade for environments in which passing or over-

taking a downstream worker is an option. While this may be an unrealistic assumption for traditional assembly lines with a single piece of equipment, there are many situations in which this may be the more appropriate assumption, such as warehouses (Bartholdi and Eisenstein, 1996), and systems with general purpose equipment. Throughout this section, we consider such environments and assume that workers are allowed to pass each other and not get blocked by the downstream worker occupying a station. To provide intuition on our analysis, we begin with a case study.

#### 3.1. Case study: $v_1$ and $v_2$ constant, $v_1 < 1 < v_2$

We define  $q_B(t)$  ( $q_A(t)$ ) as the position of worker B (worker A) along the production line at time  $t$ . We will reset time whenever a job is finished. Note that the starting worker (B) can pass the second worker (A) for some time but if she finishes her job before A, the organization of the bucket brigade has to change (see below). Since the speed of worker B speed is equal to 1 for all portions of the work we find

$$q_B(t) = t \quad \text{for } 0 \leq t \leq 1.$$

For worker A,  $q_A(t)$  will change as a function of where he started. Let  $q^0$  denote the location that worker A took over the first job, that is,  $q^0 := q_A(0)$ . If worker A starts at a point after the break point  $X$  (i.e.,  $q^0 > X$ ), then the location of worker A at time  $t$  will be governed by

$$q_A(t) = q^0 + v_2 t \quad \text{for } 0 \leq t \leq \frac{1 - q^0}{v_2}.$$

If worker A takes over the job at some point before  $X$  (i.e.,  $q^0 \leq X$ ) then we get:

$$q_A(t) = \begin{cases} q^0 + v_1 t & \text{for } t < t_X, \\ X + v_2(t - t_X) & \text{for } t \geq t_X, \end{cases}$$

where  $t_X$  is the time it takes worker A to get to  $x = X$ :

$$t_X = \frac{X - q^0}{v_1}.$$

It takes worker A  $\bar{t}_1$  time units to complete the first job, which is found by setting  $q_A(t) = 1$ .

$$\bar{t}_1 = \begin{cases} \frac{1-q^0}{v_2} & \text{for } q^0 > X, \\ t_X + \frac{1-X}{v_2} = X\left(\frac{1}{v_1} - \frac{1}{v_2}\right) - \frac{q^0}{v_1} + \frac{1}{v_2} & \text{for } q^0 \leq X. \end{cases}$$

In general,  $\bar{t}_n$  is the amount of time that worker A spends to process job  $n$ . After the completion of a job, say job  $n$ , the new starting point for worker A is the point where worker B is at time  $\bar{t}_n$  assuming that we reset time whenever a job is finished. Hence, we end up with a piecewise linear map of the form  $q^{n+1} = f(q^n)$  given by

$$q^{n+1} := q_B(\bar{t}_n) = \begin{cases} \frac{1-q^n}{v_2} & \text{for } q^n > X, \\ X\left(\frac{1}{v_1} - \frac{1}{v_2}\right) - \frac{q^n}{v_1} + \frac{1}{v_2} & \text{for } q^n \leq X. \end{cases} \tag{1}$$

The map is piecewise linear with slopes  $-1/v_1$  and  $-1/v_2$ . The elbow of the map is at  $q^n = X$  with  $f(X) = (1 - X)/v_2$ . The map is valid for all velocity combinations of  $v_1$  and  $v_2$ .

To analyze this map further, we need a few key definitions from dynamical systems theory, and their implications for the behavior of the bucket brigade.

- For any map  $x_{n+1} = f(x_n)$  we call  $x = x^*$  a fixed point if  $f(x^*) = x^*$ . Graphically this implies that the graph of  $f(x)$  intersects the diagonal  $x_{n+1} = x_n$  at  $x^*$ . For a two worker bucket brigade a fixed point implies a starting point  $x^*$  for the second worker such that the first worker always works from 0 to  $x^*$  and the second worker from  $x^*$  to 1.
- A point  $p$  is called a period two point if  $f(f(p)) = p$ . The corresponding orbit  $p \rightarrow f(p) := q \rightarrow f^2(p) = f(q) = p \rightarrow \dots$  is called a period-two orbit. Graphically this implies that the graph of  $f^2(x)$  intersects the diagonal at two points  $p$  and  $q$  different from the fixed point. A periodic orbit in a two worker bucket brigade implies that handover of the product from the first to the second worker happens alternately at position  $p$  and at position  $q$ .
- A fixed point  $x^*$  is called (asymptotically) stable if there exists a neighborhood of the fixed point such that all initial conditions inside this neigh-

borhood approach the fixed point under the iterations of the map  $f(\cdot)$ . It is a theorem in dynamical systems that  $x^*$  is stable if the slope  $|f'(x^*)| < 1$  (Alligood et al., 1996). A fixed point is globally stable if all initial conditions get attracted to it. If the fixed point is globally stable, then the bucket brigade will attract to the fixed point from any starting position. We call such a system self-balanced. Similarly, a period-two orbit  $p \rightarrow q \rightarrow p$  is stable if  $p$  and  $q$  are stable fixed points of  $f^2(\cdot)$ . We call a bucket brigade that has a stable period two-orbit “self-organized”.

The most important feature of the map (1) is that there always exists a fixed point  $x^*$  where  $f(x^*) = x^*$ . We now restrict the analysis to  $v_1 < 1 < v_2$ . Then, if  $X \leq x^*$ , the fixed point is always stable since the absolute value of the slope at  $x^*$ ,  $|-1/v_2| < 1$ . From (1), we can calculate

$$x^* = \frac{1}{1 + v_2}. \tag{2}$$

If on the other hand,  $X > x^*$ , the fixed point

$$x^* = \frac{X(v_2 - v_1) + v_1}{v_2(1 + v_1)} \tag{3}$$

is unstable.

Fig. 1 shows all four possible cases for the dynamics of the bucket brigade. Each of the sub-figures shows: (i) the Poincaré map,  $q^{n+1} = f(q^n)$  (depicted by the dashed line), (ii)  $q^{n+2} = f^2(q^n) = f(f(q^n))$ , and (iii)  $y = x$  line as the diagonal. Fig. 1(a) and (b) shows the case of  $X \leq x^*$ , which results in a stable fixed point. Fig. 1(c) and (d) shows the case of  $X > x^*$ , which results in an unstable fixed point. The critical quantity for the dynamics in both cases is the time that it takes worker A to get through the entire line, which we denote as  $f(0)$ . Then,

$$f(0) = \frac{X}{v_1} + \frac{1-X}{v_2}. \tag{4}$$

Note that  $1/f(0)$  is the average velocity of worker A,  $\bar{v}_A$ . Hence, if  $f(0) > 1$  (or equivalently,  $\bar{v}_A < 1$ ), then worker B is said to be faster on average than worker A. We can distinguish four cases, given by the relationship between  $X$  and  $x^*$ , and  $\bar{v}_A$

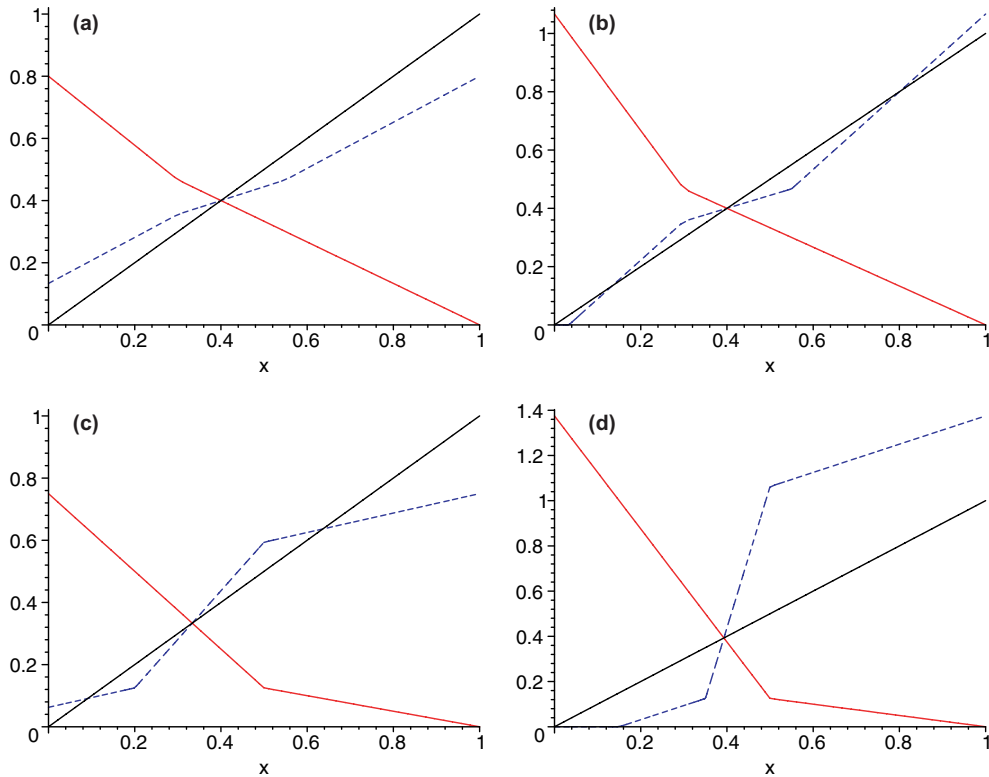


Fig. 1. All possible dynamics for  $v_1 < 1$  when passing is allowed. (a)  $X \leq x^*$ ,  $\bar{v}_A > 1$ : globally stable fixed point, (b)  $X \leq x^*$ ,  $\bar{v}_A \leq 1$ : locally stable fixed point, unstable period-two orbit, (c)  $X > x^*$ ,  $\bar{v}_A > 1$ : unstable fixed point, stable period-two orbit, (d)  $X > x^*$ ,  $\bar{v}_A \leq 1$ : globally unstable fixed point, reversal of workers.

and 1. These cases can easily be understood by studying Fig. 1, and are rigorously proved in Section 3.2. The following summarizes the dynamics in each of the four cases.

- $X \leq x^*$ , stable fixed point:
  - Fig. 1(a) shows the case  $\bar{v}_A > 1$ . We see that the stable fixed point attracts all initial conditions.
  - Fig. 1(b) shows the case  $\bar{v}_A \leq 1$ . In this case, the fixed point is still stable, but there also exists a period-two orbit that is unstable. That is, all initial conditions inside the period-two orbit converge to the fixed point and all initial conditions outside the period-two orbit lead to a situation where worker B finishes her job before worker A does, which violates the ordering.

- $X > x^*$ , unstable fixed point:
  - Fig. 1(c) shows the case  $\bar{v}_A > 1$ . We see that the fixed point is unstable. However, there exists a stable period-two orbit that attracts all initial conditions and preserves the order of the bucket brigade.
  - Fig. 1(d) shows the case  $\bar{v}_A \leq 1$ . In this case, the fixed point is unstable and all initial conditions eventually lead to a situation where worker B finishes her job before worker A thus violating the ordering.

### 3.2. General results for the passing case

The previous case study can be generalized into theorems. In the following, we assume that worker A has constant speeds  $v_1$  on the interval  $[0, X]$  and  $v_2$  on the interval  $[X, 1]$ . Theorem 2 extends the re-

sults to the case of a general velocity function with a single breakpoint,  $0 < X < 1$ . The term slower worker (faster worker) refers to the worker that is slower (faster) *on average*. The speeds  $v_1$  and  $v_2$  refer to the speed ratio between a reference worker and a locally faster or slower worker. The speeds  $v_1$  and  $v_2$  are not restricted and hence we can choose to have our reference worker (the one with speed 1) to always be the starting (i.e., upstream) worker.

**Lemma 1.** *There exists a unique fixed point that balances the bucket brigade for all values of  $v_1$  and  $v_2$ .*

**Proof.** Consider a starting point of the second worker at  $x = 0$ . Then the next handover point is given by  $f(0)$ . If the second worker is faster on average than the first worker then  $0 \leq f(0) \leq 1$ . If the second worker is slower on average, then there exists a starting point  $q$  such that the first and the second worker finish at the same time, i.e.,  $f(q) = 1$ . Since  $f(1) = 0$  in all cases, and since

$f$  is continuous, by the Intermediate Value Theorem there exists a point  $x^*$  such that  $f(x^*) = x^*$ . Finally, since  $f$  is always monotonously decreasing, the fixed point is unique.  $\square$

**Lemma 2.** *Consider the parameter space  $(v_1, v_2, X)$ . The manifold  $X = 1/(1 + v_2)$  separates stable from unstable fixed points.*

**Proof.** The fixed point on one part of the piecewise linear Poincaré map is given by  $x^* = 1/(1 + v_2)$  (Eq. (2)). The corner of the piecewise linear map is at  $x = X$  and hence the fixed point moves from a stable region to an unstable region. Note that the slopes of the Poincaré maps are given by  $-1/v_1$  and  $-1/v_2$  (see (1)) and therefore the maps oscillate around the fixed points.  $\square$

The following theorem divides the parameter space  $(v_2, X)$  into four regions with distinct dynamical behavior. Fig. 2 shows the parameter space for (a)  $v_1 < 1$ , (b)  $v_1 > 1$ .

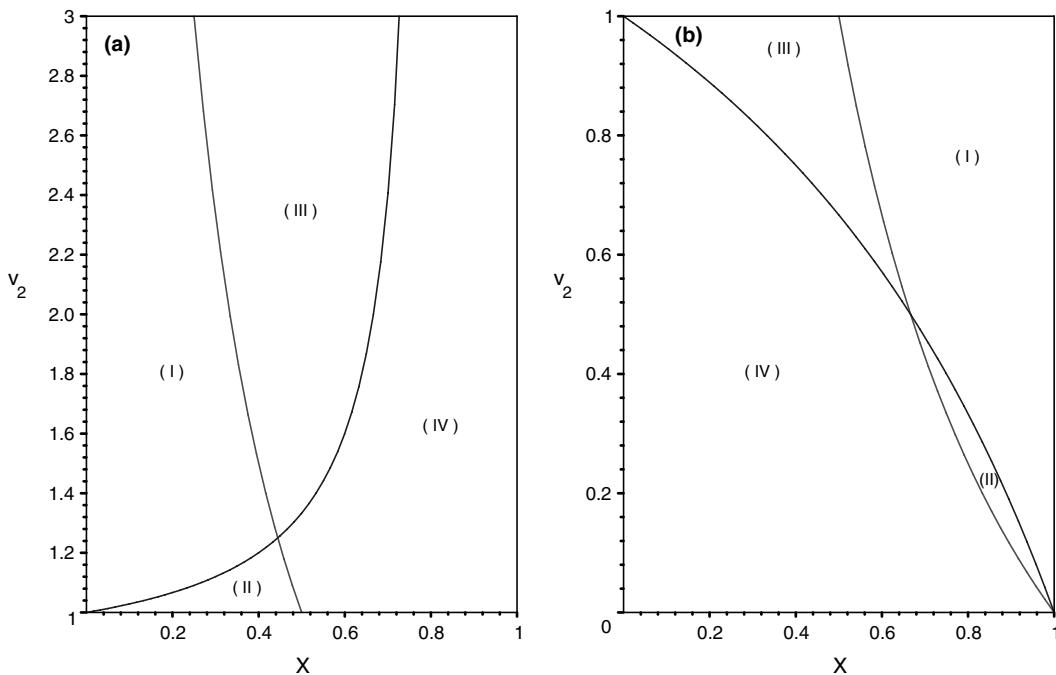


Fig. 2. Parameter space  $v_2, X$  for (a)  $v_1 < 1$ , (b)  $v_1 > 1$ .

**Theorem 1.** The two curves  $f(0) = 1$  and  $X = 1/(1 + v_2)$  determine the average velocity of the second worker and the stability of the fixed point. They intersect in parameter space  $(v_2, X)$  and generate four regions with different dynamical behavior. The resulting dynamics can be summarized as follows:

1. Region (I): If  $f(0) < 1$  (i.e.,  $\bar{v}_A > 1$ ) then every locally stable fixed point is also globally stable (Fig. 1(a)).
2. Region (II): If  $f(0) \geq 1$  (i.e.,  $\bar{v}_A \leq 1$ ) and the fixed point is locally stable, then there exists an unstable period-two orbit. All initial conditions inside the period-two orbit lead to a balanced line at the fixed point, initial conditions outside the period-two orbit lead to a reversal of the finishing order of the bucket brigade (Fig. 1(b)).
3. Region (III): If  $f(0) < 1$  (i.e.,  $\bar{v}_A > 1$ ) and the fixed point is locally unstable, then there exists a stable period-two orbit that attracts all initial conditions (Fig. 1(c)).
4. Region (IV): If  $f(0) \geq 1$  (i.e.,  $\bar{v}_A \leq 1$ ) then every locally unstable fixed point leads to a reversal of the finishing order of the bucket brigade for all initial conditions (Fig. 1(d)).

**Proof.** The proof is based on two facts illustrated in Fig. 3:

- (i) If  $f(0) < 1$  then the Poincaré map  $f(x)$  is defined for all initial conditions and in particular  $0 < f(0) < 1$  while  $f(1) = 0$ .
- (ii) Recall that  $x = X$  is the point where the velocity order switches. We find that  $f^2(X) > X$  and  $f^2(Y) < Y$ , where  $Y$  is the preimage of  $X$ . If the fixed point is stable then  $x^* > X$  and since the map  $f$  alternates the orbits around the fixed point,  $Y > X$ . If the fixed point is unstable, then  $x^* < X$  and hence,  $X > Y$ . The second iteration of the Poincaré map is piecewise linear and consists of three line segments with the slopes  $(v_1 v_2)^{-1}, v_2^{-2}, (v_1 v_2)^{-1}$ , respectively. The end points of these line segments are  $f^2(0), f^2(X), f^2(Y)$  and  $f^2(1)$  (see Fig. 1).

Simple geometry and the Intermediate Value Theorem implies all the statements. The proof for the

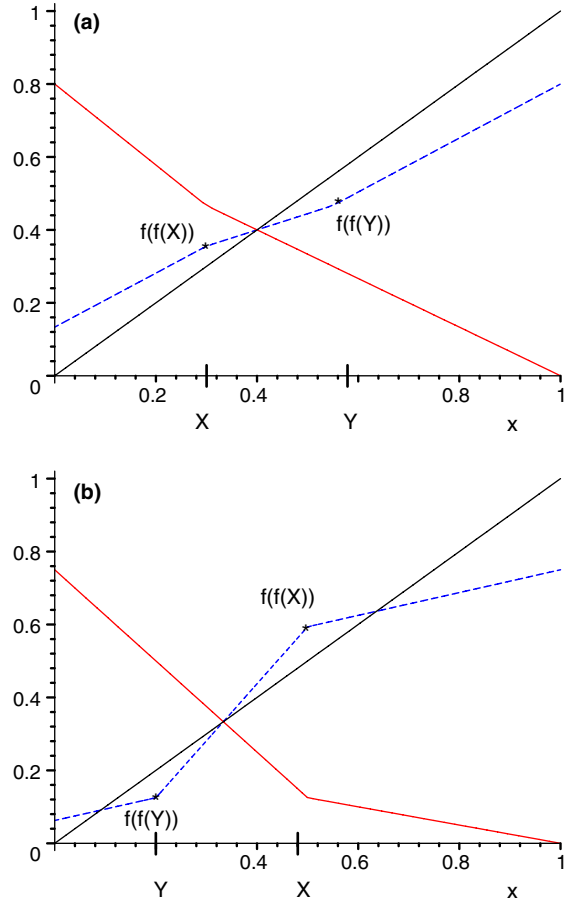


Fig. 3. Points  $X, Y, f^2(X), f^2(Y)$  in Fig. 1(a) and (c). (a)  $X \leq x^*, \bar{v}_A > 1$ : globally stable fixed point, (b)  $X > x^*, \bar{v}_A > 1$ : unstable fixed point, stable period-two orbit.

case  $f(0) > 1$  works in the same way by replacing 0 with the position  $q$  for which  $f(q) = 1$  (compare the proof of Lemma 1).  $\square$

**Theorem 2.** All previous lemmas and theorems are still valid if the ratio of the velocities in  $[0, X)$  and  $[X, 1]$  are not piecewise constant but depend on the exact location along the production line, as long as there is only one point at which the relative order of the workers' velocities changes.

**Proof.** Assume a speed distribution of the form

$$v(x) = \begin{cases} v_1(x) & \text{for } x < X, \\ v_2(x) & \text{for } x \geq X. \end{cases} \tag{5}$$

where either  $v_1(x) < 1$  for all  $x < X$  and  $v_2(x) > 1$  for all  $x \geq X$  or  $v_1(x) > 1$  for all  $x < X$  and  $v_2(x) < 1$  for all  $x \geq X$ . The position of the second worker is given through the implicit solution of the differential equation  $q'(t) = v_1(q(t))$ . For  $q_0 < X$  we get

$$\int_{q_0}^{q(t)} \frac{d\xi}{v_1(\xi)} = t \quad \text{for } t < t_X,$$

$$\int_X^{q(t)} \frac{d\xi}{v_2(\xi)} = t - t_X \quad \text{for } t \geq t_X,$$

where  $t_X$  is the solution of the equation  $\int_{q_0}^X \frac{d\xi}{v_1(\xi)} = t_X$ . The finishing time  $\bar{t}$  is similarly given as the solution of the equation  $t_X + \int_X^1 \frac{d\xi}{v_2(\xi)} = \bar{t}$  for  $q_0 < X$  and  $\int_{q_0}^1 \frac{d\xi}{v_2(\xi)} = \bar{t}$  for  $q_0 > X$ . The Poincaré map  $q^{n+1} = f(q^n)$  is still given as

$$q^{n+1} := q_B(\bar{t}_n) = \bar{t}_n. \tag{6}$$

Note that

- $f(0) = \int_0^X \frac{d\xi}{v_1(\xi)} + \int_X^1 \frac{d\xi}{v_2(\xi)}$  is still  $1/\bar{v}_a$ , the inverse of the average velocity for worker A. Hence, if  $\bar{v}_A > 1$ , then  $f(0) < 1$ , whereas  $f(0) > 1$ , implies worker B is faster than worker A on average. Since  $f(1) = 0$ , the arguments for Lemma 1 still hold.
- Uniqueness and stability of the fixed point can easily be determined by noting that

$$\frac{d\bar{t}}{dq^n} = \begin{cases} -\frac{1}{v_1(q^n)} & \text{for } q^n < X, \\ -\frac{1}{v_2(q^n)} & \text{for } q^n > X. \end{cases} \tag{7}$$

Hence the Poincaré map is monotonously decreasing and fixed points change stability as they move across the break point  $X$ . This proves the generalization of Lemma 2.

The geometric arguments of Theorem 1 continue to hold if the Poincaré map is not piecewise linear but is only monotonously decreasing.  $\square$

**Lemma 3.** *The throughput for a period-two orbit is always less than the throughput of the fixed point.*

**Proof.** By construction of the Poincaré map (6) we have that  $f(q) = \bar{t}$ , the time between successive finishes. Hence, to prove the theorem we need to show that for a periodic orbit  $p_1 \rightarrow p_2 \rightarrow p_1 \dots$  it takes longer to finish the periodic orbit than to finish two products when starting at the fixed point, i.e.,  $p_1 + p_2 > 2x^*$ .

Consider Fig. 1(c). Since  $f(p_1) = p_2$ , the interval  $[p_1, x^*]$  is mapped to the interval  $[x^*, p_2]$  by  $f$ . However,  $f$  is piecewise linear with a slope less than  $-1$  on that interval and hence  $x^* - p_1 < p_2 - x^*$ , which proves the claim. A similar argument holds for an unstable periodic orbit as can be seen in Fig. 1(b).  $f$  is contracting on the interval  $[x^*, p_2]$  and hence the statement again holds.  $\square$

The above general results imply important practical considerations for the design and management of bucket brigade lines. We discuss these issues further in Section 5. We next describe a novel rule that significantly improves the performance of an unstable bucket brigade without any managerial intervention. The only requirement is that workers are allowed to pass each other, which as we discussed above, is possible in quite a number of manufacturing environments.

### 3.3. A new rule that yields self-organization

Our current rules assign a fixed starting worker and a fixed finishing worker. However, for some speed configurations the starting worker will finish first and hence, the line becomes unbalanced. Note that this situation would never arise under the original assumptions of Bartholdi and Eisenstein (1996) where the slowest to fastest ordering does not change throughout the entire line.

We augment our bucket brigade rules and stipulate that if the starting worker finishes first, the workers will reverse their order and start following the bucket brigade rules according to this new ordering. This rule leads to the following lemma.

**Lemma 4.** *Reversing the worker order for a given set of velocities and a given break point interchanges the regions as follows: region (I) in Fig. 2(a) maps to region (IV) in Fig. 2(b) and region (II) in Fig. 2(a) maps to region (III) in Fig. 2(b), and vice versa.*

**Proof.** A reversal of worker order for given functions  $v_1(x)$ ,  $v_2(x)$  and  $X$  is equivalent to renaming the second worker (i.e., worker A) as the reference worker (i.e., worker B). This implies that the dynamics of the reorganized bucket brigade can be found from Theorem 1 by setting  $v_1(x) = 1/v_1(x)$  and  $v_2(x) = 1/v_2(x)$  and having a new break point  $Y$  given by the solution of  $X = \int_0^Y \frac{1}{v_1(x)} dx$ , which gives  $Y = v_1 X$  for constant  $v_1$ .  $\square$

To illustrate the consequences of this result, we consider the following. Assume  $\bar{v}_A < 1$ . By Theorem 1 we know that if the fixed point is unstable all initial conditions will lead to reversals. If we now switch the positions of the workers as the rule implies, the faster worker is now at the end and the fixed point becomes globally stable. Hence, the bucket brigade balances to a fixed point upon reversal of worker orders. In Section 5, we will show that achieving self-balancing behavior, while highly desirable from a managerial point-of-view, may actually lead to significant losses in throughput performance.

#### 4. Dynamics for the blocking case

In this section we assume that workers cannot pass each other. That is, a worker becomes blocked if he/she catches up with the downstream worker. This is the original assumption used by Bartholdi and Eisenstein (1996) to study apparel manufacturing lines and warehouses. For many systems with specialized equipment this becomes a more appropriate assumption due to the increased costs of duplicate tooling.

Under the assumption of continuous tasks, the concept of blocking is different than usual. Rather than idling and waiting for the other worker to leave a workstation, if a faster worker is blocked behind another (slower) worker, then he/she is forced to work at the speed of the leading worker. The basic assumption is that both workers can get arbitrarily close and still work on their own tasks. Having stated this difference, we note that the analysis of dynamics of systems with discrete tasks yields similar results and is omitted for brevity.

**Lemma 5.** For general velocity  $v_1(x) < 1$ , blocking occurs for initial conditions  $q < q_0$ , where  $q_0$  is given by the solution of  $X = \int_{q_0}^X \frac{1}{v_1(x)} dx$ . For  $v_1(x) > 1$  blocking occurs for  $q < q_0$ , where  $q_0$  is given by the solution of  $1 = \int_{q_0}^X \frac{1}{v_1(x)} dx + \int_X^1 \frac{1}{v_2(x)} dx$ . In particular, for constant  $v_1 < 1$ , blocking occurs whenever  $q_0 < X(1 - v_1)$ , for constant  $v_1 > 1$ , blocking occurs for  $q_0 < \frac{v_1}{v_2}(1 - X) + X - v_1$ .

**Proof.** Blocking occurs if worker B arrives at  $X$  earlier than worker A, which implies the result.  $\square$

**Corollary.** The fixed point never gets blocked.

**Proof.** By definition, the fixed point is the point where worker B is when worker A finishes, which implies the corollary.  $\square$

**Theorem 3.** There exist three qualitatively different regimes for the blocking case bounded by the curves  $f(0) = 1$  and  $X = 1/(1 + v_2)$  of Theorem 1.

1. Region (I): The fixed point is globally stable (Fig. 4(a)).
2. Region (II): A locally stable fixed point is surrounded by an unstable periodic orbit surrounded by a stable periodic orbit. The stable periodic orbit involves blocking at every second iteration (Fig. 4(b)).
3. Region (III): An unstable fixed point is surrounded by a globally stable periodic orbit. The periodic orbit may or may not involve blocking at every second iteration (Fig. 4(c)).

Fig. 5(a) and (b) shows the parameter space  $(v_2, X)$  and the three regions with different dynamical behavior for  $v_1 < 1$  and  $v_1 > 1$  cases. Regions (I) and (II) are identical to the Regions (I) and (II) in the passing case, Region (III) is the union of Regions (III) and (IV) of the passing case. Since the fixed points are never blocked, the curve  $X = 1/(1 + v_2)$  still separates stable from unstable fixed points. The second large periodic orbit results from the fact that in the blocking case, the worker order is always preserved and hence  $f(0)$  is always bounded by 1. Further details of the proof are very

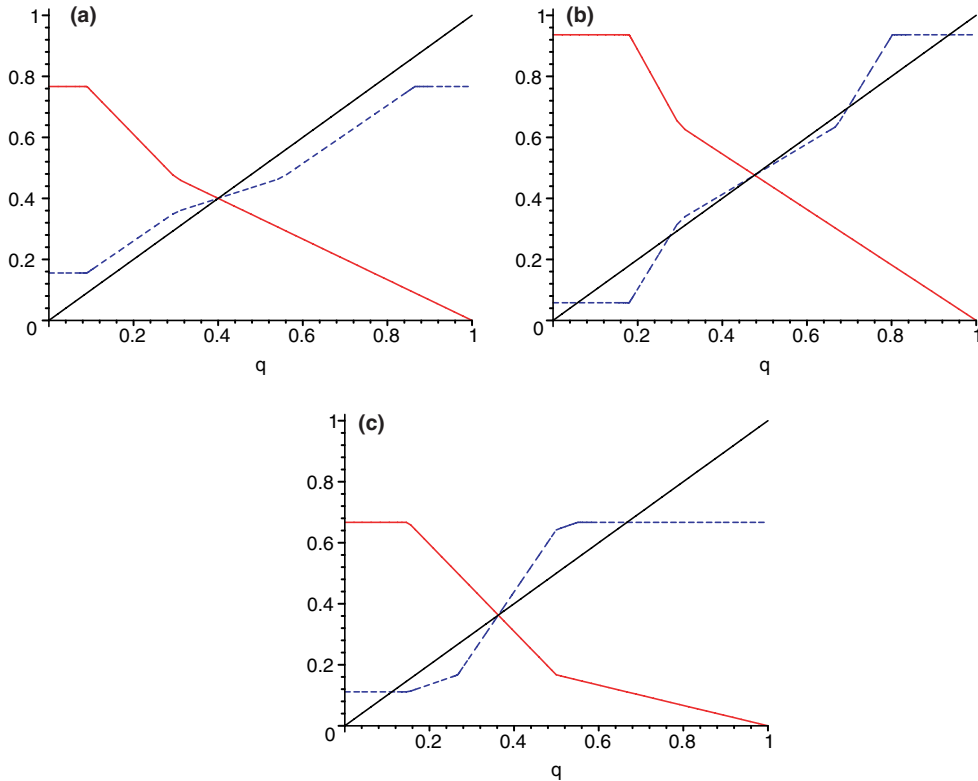


Fig. 4. All possible dynamics for  $v_1 < 1$  under the blocking assumption. (a) Globally stable fixed point, (b) bistability: both the fixed point and the large periodic orbit are stable, (c) unstable fixed point, globally stable periodic orbit.

similar to that given in the proof of Theorem 1, and hence are omitted for brevity.

**5. Insights and operating principles**

So far, we have been interested in the dynamical behavior of the bucket brigade under different conditions. Given that different operating conditions lead to different dynamics, it is useful to evaluate the performance of the bucket brigade in each case and identify the tradeoffs that may provide guidelines for practitioners considering implementing bucket brigades. We are mainly interested in two types of performance measures for bucket brigades: (1) throughput, and (2) self-balancing or self-organizing behavior. Note that for cases in which the dynamics converges to a period-two orbit rather than a fixed point, we call the bucket

brigade to be “self-organizing” rather than “self-balancing”.

Obviously, the most preferable region with respect to both of these performance measures is Region (I) for both passing and blocking environments, since the bucket brigade is self-balancing with optimal throughput. However, given that the regions are determined by the parameters  $v_1$ ,  $v_2$  and  $X$ , what are some operating principles and guidelines for systems in Regions (II), (III) and (IV)?

An important point to note before this discussion is that the implementation of bucket brigades is most appealing in environments in which determination of these parameters (namely,  $v_1$ ,  $v_2$  and  $X$ ) is costly or difficult. Hence, these parameters may not be available. The following discussion, however, makes a case for at least a rough-cut determination of these parameters. Note that all

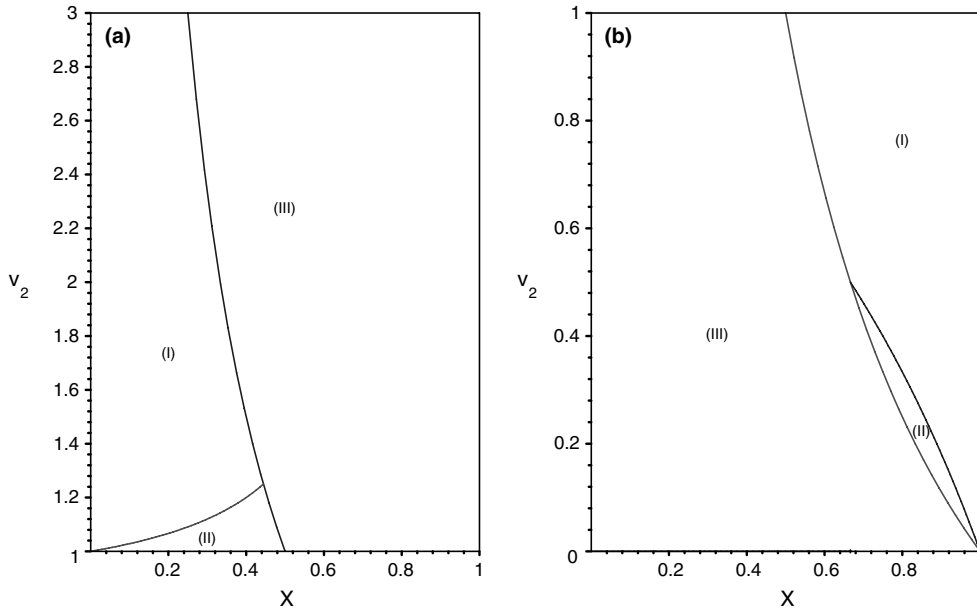


Fig. 5. Parameter space  $v_2, X$  for (a)  $v_1 < 1$ , (b)  $v_1 > 1$ .

the following discussions hold qualitatively for non-constant velocities as long as there exists only one point at which the relative order of the workers' velocities changes. However, then the curves separating these regions cannot be determined analytically any more.

*5.1. Performance in Region (II)*

A two-worker bucket brigade system is said to be in Region (II) if the second worker is slower on average than the starting worker and if

- (a) the break point is small enough ( $X < 1/(1 + v_2)$ ) and the second worker is faster than the starting worker after the break point, or
- (b) the break point is large enough ( $X > 1/(1 + v_2)$ ) and the second worker is slower than the starting worker after the break point.

As discussed above, in Region (II) we have a locally stable fixed point, meaning that if we start close enough to the fixed point, the bucket brigade is self-balancing. Outside this basin of attraction, the bucket brigade will not be attracted to the fixed point and will depict different behavior in the pass-

ing and blocking environments: (1) If passing is allowed, then the reference worker (who is always assumed to be upstream) will finish before worker A, leading to a reversal of the worker order. (2) If passing is not allowed, the bucket brigade will attract to a stable periodic orbit, which involves blocking at every second iteration.

While the fixed point is not globally attractive, to achieve self-balancing and optimal throughput, starting close enough to the fixed point is sufficient in Region (II). Hence, it may be worthwhile to spend the effort to roughly estimate the system parameters to initialize the bucket brigade inside the basin of attraction, so that self-balancing behavior with optimal throughput is enabled.

*5.2. Performance in Region (III)*

A two-worker bucket brigade system is said to be in Region (III) if the second worker is faster on average than the starting worker and if

- (a) the break point is large enough ( $X > 1/(1 + v_2)$ ) and the second worker is faster than the starting worker after the break point, or

- (b) the break point is small enough ( $X > 1/(1 + v_2)$ ) and the second worker is slower than the starting worker after the break point.

In both passing and blocking environments, a bucket brigade in Region (III) self-organizes to a period-two orbit. Given that the bucket brigade does not self-balance (to a fixed point) is there any losses in the throughput performance of the bucket brigade? In the following discussion, we compare the throughputs of the self-organized bucket brigade in the passing and blocking environments with that of the fixed point for a particular case of  $X = 1/2$ . Note that the throughput of the fixed point represents the performance of a traditional static worker allocation policy, where each worker is assigned to do a portion of the line. While unstable, the throughput of the fixed point (which is the same for the passing and blocking cases) will be the highest for a given worker order, and can be calculated as below:

$$TP_{fp} = \frac{1}{x^*} = \frac{2v_2(1 + v_1)}{v_1 + v_2}. \tag{8}$$

It is straightforward to evaluate the throughput of the self-organized bucket brigade (i.e., periodic orbit) under the passing and blocking assumptions. Using the results from Section 3.1, the throughput of the stable periodic orbit for the passing case is equal to

$$TP_{pass} = \frac{2}{p_1 + p_2} = \frac{4v_2(v_1v_2 - 1)}{3v_1v_2 - 3v_2 - v_1 + v_2^2}, \tag{9}$$

since two jobs will be completed in every cycle. Using the same techniques, we can derive the throughput for the stable periodic orbit under the blocking case. The throughput of the bucket brigade in the blocking environment will depend on whether the bucket brigade incurs any blocking or not. It is straightforward to show that no blocking occurs if  $v_1 > \frac{v_2^2 - v_2 + 1}{v_2^2}$ . In this region, the dynamics (and hence, the throughput) is identical to that in the passing environment. For  $v_1 \leq \frac{v_2^2 - v_2 + 1}{v_2^2}$ , the reference worker is sufficiently faster than worker A in  $[0, 1/2)$  and catches up with the downstream worker, which results in blocking. The throughput in this case can be calculated by

determining the length of a cycle and noting that there are two job completions per cycle. Hence, the throughput of the stable periodic orbit under the blocking case is given by the piecewise function

$$TP_{block} = \begin{cases} \frac{4v_1v_2^2}{2v_1v_2 + v_2^2 - v_1 - v_2 + 1} & \text{for } v_1 \leq \frac{v_2^2 - v_2 + 1}{v_2^2}, \\ \frac{4v_2(v_1v_2 - 1)}{3v_1v_2 - 3v_2 - v_1 + v_2^2} & \text{for } v_1 > \frac{v_2^2 - v_2 + 1}{v_2^2}, \end{cases} \tag{10}$$

To compare the performances under the passing and blocking assumptions we choose  $X = 1/2$  and  $(v_1, v_2)$  in Region III which implies  $v_1 < 1$ . Fig. 6(a) graphs  $TP_{block} - TP_{pass}$  and Fig. 6(b) shows  $TP_{fp} - TP_{block}$  for  $\{(v_1, v_2) | 0.6 \leq v_1 \leq 0.99, f(0) < 1\}$ . We make the following observations:

- Obviously the throughput of the stable periodic orbit in the blocking case is equal to the passing case when there is no blocking. However, with blocking (i.e., small values of  $v_1$ , the throughput is higher for the blocked periodic orbit than for the unblocked periodic orbit. In any case, the difference of the throughputs for the two periodic orbits is small.
- As expected, the throughput of the fixed point is higher than that for the periodic orbit under both passing and blocking assumptions. For small  $v_1$  the unstable fixed points has a significantly higher throughput than the stable periodic orbit.

The first observation leads to the counterintuitive situation that a system that slows down a worker for a certain time (i.e., due to blocking) has higher throughput than a system that allows the same workers to always work at their own speed and pass each other. The reason for this can be understood by inspection of the behavior of the periodic points as  $v_1$  decreases. In the blocking case, the periodic points do not depend on  $v_1$  while in the passing case we have that for decreasing  $v_1$ ,  $p_1 \rightarrow 0$  and  $p_2 \rightarrow 1$ . This forces the worker A to work on a large part of the production line where he is very slow. Note that this result also implies that under

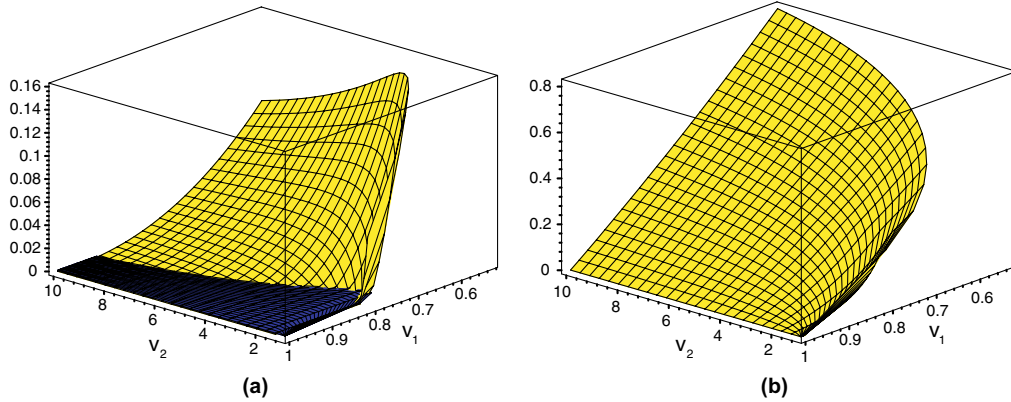


Fig. 6. (a)  $TP_{\text{block}} - TP_{\text{pass}}$  (b)  $TP_{\text{fp}} - TP_{\text{block}}$  on a grid of  $v_1$  and  $v_2$ .

this type of speed structure, the investment of a dedicated production line for each worker (to allow passing) will not necessarily improve the overall throughput of the bucket brigade.

For systems in this Region (III), managers need to consider the tradeoff between the throughput and self-organizing behavior. If throughput gain outweighs the gains due to self-organization, then a traditional allocation of workers to the portions of the line (represented by the fixed point) is a better option to the use of bucket brigades.

5.3. Performance in Region (IV) in the passing environment

In the passing environment, a two-worker bucket brigade system is said to be in Region (IV) if the second worker is slower on average than the starting worker and if

- (a) the break point is large enough ( $X > 1/(1 + v_2)$ ) and the second worker is faster than the starting worker after the break point, or
- (b) the break point is small enough ( $X < 1/(1 + v_2)$ ) and the second worker is slower than the starting worker after the break point.

Region (IV) in the passing environment poses some interesting dilemma; the fixed point clearly creates maximal throughput but it is not attracting. Hence no self-balancing occurs. However, changing the order of the workers (i.e., reversal)

will yield a globally attracting fixed point, since Region (IV) maps to Region (I) by Lemma 4. This fixed point has the maximal throughput for the given worker order, but is this maximal throughput higher than the throughput that one would get for a traditional static work allocation at the original unstable fixed point?

**Theorem 4.** For  $v_1 < 1$  ( $v_1 > 1$ ) and  $(v_2, X)$  in Region (IV) the throughput of the unstable fixed point is always higher (lower) than the throughput attainable with a reversed worker order.

**Proof.** For  $v_1 < 1$  the throughput at the unstable fixed point is given by  $TP_u = \frac{v_2(1+v_1)}{X(v_2-v_1)+v_1}$ . Reversing the worker order leads to a stable fixed point with throughput  $TP_s = 1 + v_1$ . It is easy to show that  $TP_u > TP_s$  for  $v_2 > v_1$  and  $0 < X < 1$ .

For  $v_1 > 1$  the throughput of the unstable fixed point is  $TP_u = 1 + v_2$ . Reversing the worker order leads to a stable fixed point with throughput  $TP_s = \frac{v_1(1+v_2)}{X(v_1-v_2)+v_1v_2}$ . It is easy to show that  $TP_u < TP_s$  in region (IV) of Fig. 2(b). □

**Corollary 1.** Reversal of worker ordering in Region (IV) of Fig. 2(b) leads to a self-balancing bucket brigade with optimal throughput relative to any worker order.

**Corollary 2.** Bucket brigades are not useful in the following regions: Region (IV) of Fig. 2(a) and Region (I) of Fig. 2(b).

Corollaries 1 and 2 have important practical implications. If  $v_1 > 1$  and the system belongs to Region (IV) (i.e.,  $X < 1/(1 + v_2)$  and  $\bar{v}_A < 1$ ), starting the bucket brigade with worker A at the end will not be self-organizing since the reference worker will finish before worker A. Implementing the new rule discussed in Section 3.3 will reverse the order of the workers. Upon this reversal, as Corollary 1 shows, the system will self-balance to a fixed point, with optimal throughput. Clearly, this is a highly desirable case in which the bucket brigade requires almost no management intervention; by simply implementing the reversal rule, the bucket brigade becomes self-balancing and has improved throughput performance.

Consider, however, a system for which  $v_1 < 1$  and again, the system belongs to Region (IV) (i.e.,  $X > 1/(1 + v_2)$  and  $\bar{v}_A < 1$ ). Note that now, we are referring to the parameter space shown in Fig. 2(a). In this case, again, starting the bucket brigade with worker A at the end will result in the reference worker finishing before worker A. If the reversal rule is applied, the bucket brigade is now considered to be in Region (I) of Fig. 2(b). We know from the above discussion that, while the bucket brigade becomes self-balancing to a fixed point, the throughput of this fixed point is inferior to that of the unstable fixed point of the original ordering of the workers. In other words, if the throughput performance is the main concern, one will be better off by implementing a traditional static allocation policy rather than a bucket brigade.

#### 5.4. Guidelines for cross-training

Our results show that when workers are better or faster in different regions of the production line, the dynamical behavior changes. An important question is to understand how one can train workers so that the bucket brigade results in a self-balancing line with optimal throughput (i.e., globally stable fixed point, Region (I)).

For this purpose, Figs. 2 and 5 provide useful guidelines for passing and blocking environments, respectively. To demonstrate, let's consider the passing environment. In Fig. 2(a) (i.e.,  $v_1 < 1$  case), moving from Region (III) to Region (I) means that

the bucket brigade becomes self-balancing at the fixed point, rather than converging to a periodic-orbit. Note that what divides Regions (I) and (III) in Fig. 2(a) is the line  $X = 1/(1 + v_2)$ . In other words, the same bucket brigade (i.e., same  $v_1$  and  $v_2$  for worker A) will result in self-balancing behavior, rather than a periodic-orbit, if the breakpoint,  $X < 1/(1 + v_2)$ . For a real life system, one way to accomplish this is to train worker A such that his speed increases to at least  $v_2$  in the interval  $[1/(1 + v_2), X]$ . In Fig. 2(b) (i.e.,  $v_1 > 1$  case), we see a similar opportunity. Again, moving from Region (I) to (III) requires increasing the speed of worker A to at least  $v_1$  in the interval  $[X, 1/(1 + v_2)]$ . Similarly, moving from Region (II) to Region (I) involves a combination of changing the break point and increasing the speed  $v_2$ .

The above are clearly situations in which more training or additional equipment to speed-up worker A may be justified due to improved performance in self-organization as well as the throughput of the line. Note, however, that the above guidelines use information on  $X$ ,  $v_1$  and  $v_2$ , which again makes a case for collecting at least some rough estimates of the worker speeds in different portions of the production line.

## 6. Conclusions and further work

We have discussed the most fundamental cases of a bucket brigade with workers having different skills at different parts of a production line, leading to different worker speeds along the line. We find that, if we implement a simple rule (i.e., switch the order of the workers if one worker passes another) the bucket brigade always self-organizes itself. We have shown however, that it may not always balance itself on fixed point but may also self-organize to a stable period-two orbit. That is, workers hand over jobs at exactly two fixed locations that they visit periodically. We found that the crucial parameter is the relative average velocity  $\bar{v}_A$  of the two workers and that the overall throughput of the line is optimal for the fixed point, even when that point is unstable.

The above discussion clearly shows that some amount of management intervention is necessary

to ensure effectiveness of bucket brigades. At the very minimum, management should assess whether worker speeds uniformly dominate each other, so that workers can in fact be ordered from slowest to fastest. In particular, any special equipment on which some workers are faster than others is an indication that implementation of bucket brigade rules may yield problematic behavior: the dynamics may still yield self-balance (or self-organization at the minimum) but the throughput performance may suffer compared to a traditional static allocation. For some manufacturing environments this may still be preferable, but managers need to be aware of this important tradeoff.

There are some obvious extensions that we plan to address in our future work in this area:

- What happens if worker A has more than two different speeds, corresponding to his different skills at more than two machines? We have not analyzed throughput and the issue of balance of the line in detail yet. However, clearly all maps relating the next starting point to the previous one are always monotonically non-increasing. Hence there is always a fixed point which may or may not be unstable. If the fixed point is unstable, then there exists always at least one stable periodic orbit. Hence nothing more complicated dynamically than a period-two orbit can happen.
- The combinations of relative speeds increases combinatorically if the number of workers increases. Hence, a complete analysis is not feasible. However, we are interested in finding general organizing principles for bucket brigades with task dependent worker speeds. For example,
  - What is the most complicated dynamics for a line with  $n$  workers and  $m$  different speed levels? In particular, is chaotic dynamics possible?
  - Is an ordering analogous to the two-worker line according to average throughput stabilizing?
  - What happens to a regular (i.e., constant speed along the production line) bucket brigade that is ordered from slowest to fastest

when there are two neighboring workers in the middle of the line that are like our two workers, A and B? Under what circumstances they do or don't "disturb" the entire bucket brigade?

- Assume that there is a specialized station in the middle of the line, at which only one worker was sufficiently specialized to perform the operations in a fast enough pace. In such a case, should one employ a single bucket brigade, two separate bucket brigades with a stationed worker in the middle or drop the bucket brigade policy altogether?
- We are also interested in considering other types of variations on the bucket brigade policy, especially when processing times are random, rather than deterministic. For example, when blocked, one could assume that a worker abandons the workpiece as work-in-process inventory and returns upstream to start another job. The job can then be taken up by the downstream worker at a later time, when he/she becomes available. This kind of policy will also have the advantage of helping to keep the workers in their most advantageous zones in the presence of uncertainty in the processing times.
- Our analysis provides a way to model and understand bucket brigade systems with high labor turnover. In particular, where in the bucket brigade should we place a worker that is just at the beginning of the learning curve? What happens as workers learn and their speeds change dynamically over time?

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