# Estimating Transport Coefficients in Re-entrant Factory Models

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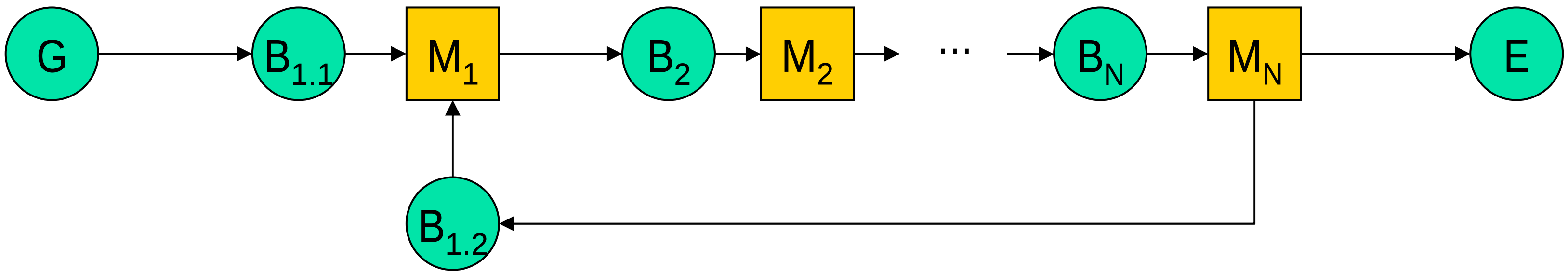
# Structure




Structure of the re-entrant manufacturing system

- G: Generator
- M: Machine
- E: Exit
- At the machine $M_N$, the particles change their attribute from 1 to 2.

# Goal

- We have the following conservation law:

$\partial_t \rho + \partial_x [V\rho - D\partial_x \rho] = 0$ , where

flux function $f(x) = V\rho - D\partial_x \rho$,

$V$: Velocity coefficient,

$D$: Diffusion coefficient,

$\rho$: Density of particles ($\rho = \rho(x,t)$).

# Goal (ctd.)

- We will try to find the *V* and *D* (velocity and diffusion coefficients of the particles in the production system) from DES (Discrete Event Simulation).
- We will use Chi Simulation-version 0.8

# Possibility 1

- $V = V(x)$ and $D = D(x)$

  $V$ and $D$ depend on $x$.

- Take a part in DES, say at the attribute $p$, and buffer $B_k$, where we have the following buffer structure:

  $B_1 M_1 \ldots B_N M_N \; B_1 M_1 \ldots B_N M_N$

# Possibility 1 (ctd.)

- Here, $x \in [0,1]$ is the position variable for the production process, formulated as follows:

$$x = \frac{k}{2N} + \frac{(p-1)}{2}, \text{ where}$$

$k$ : buffer stage,

$p$ : attribute of the loop (1 or 2),

$N$ : total # of buffers.

# Possibility 1 (ctd.)

- In general, particle $n$ is in buffer $B_k$ with attribute $p$ at time $t_{nk}^{p}$.
- Similarly, particle $n$ is in buffer $B_N$ with attribute 2 at time $t_{nN}^{2}$.
- So, $\Delta t = t_{nN}^{2} - t_{nk}^{p}$ is the time that particle $n$ takes from $B_k$ with attribute $p$, to $B_N$ with attribute 2.

# Possibility 1 (ctd.)

- Since $x = \frac{k}{2N} + \frac{(p-1)}{2}$, we get: $x = 1$ for $p = 2$ and $k = N$.

  And we get: $\Delta x = 1 - \frac{k}{2N} - \frac{p-1}{2}$.

- Average Velocity $= \frac{\text{Total Distance}}{\text{Total Time}} = \frac{\Delta x}{\Delta t}$.

  So, "estimated velocity" for particle $n$ is:

$$V_n\left(\frac{k}{2N} + \frac{p-1}{2}\right) = \frac{1 - \frac{k}{2N} - \frac{p-1}{2}}{t_{nN}^2 - t_{nk}^p}.$$

# Possibility 1 (ctd.)

- Then, for each particle n, we will find the velocity according to its position w.r.t. buffer stage and attribute: $v_1(x_{kp})$ ... $v_{100}(x_{kp})$, where $x_{kp} = \dfrac{k}{2N} + \dfrac{(p-1)}{2}$.
- Velocity coefficient: $V(x_{kp}) = \dfrac{1}{100}\sum_j v_j(x_{kp})$, from mean formula.
- Diffusion coefficient: $D(x_{kp}) = \dfrac{\frac{1}{100}\sum_j (v_j(x_{kp}) - V(x_{kp}))^2}{V(x_{kp})^2}$, from variance formula.

# Possibility 2

- $V = V(W)$ and $D = D(W)$

- $V$ and $D$ depend on Wip level of particles:

$$W(t) = \int_0^1 \rho(x,t)dx$$

# Possibility 3

- $V = V(\rho(x,t))$ and $D = D(\rho(x,t))$.
- $V$ and $D$ depend on density of particles $(\rho(x,t))$.

# Possibility 4

- $V = V(W_-(x,t), W_+(x,t))$ and $D = D(W_-(x,t), W_+(x,t))$

- $V$ and $D$ depend on right and left Wip levels of particles.

  where left Wip level is $W_-(t) = \int_0^{x} \rho(y,t)dy$

  and right Wip level is $W_+(t) = \int_x^1 \rho(y,t)dy.$

# Possibility 4 (ctd.)

- We can get the Wip level of each particle at each time.
- $W(t_{nk}^{p})$ can be found for $n=1:100,\ k=1:N,\ p:1:2.$
- We know that $V_n(x_k^p)=\dfrac{1-\dfrac{k}{2N}-\dfrac{p-1}{2}}{t_{nN}^{2}-t_{nk}^{p}}$,

  so we can easily draw $W$ -$V$ graph and see how $V$ depends on $W$.

# Possibility 4 (ctd.)

For example:

$$V(0.15) = \frac{\sum_{nkp} v_n(x_k^p) \cdot \chi_{(0.1,0.2)}(W(t_{nk}))}{\sum_{nkp} \chi_{(0.1,0.2)} W(t_{nk})}$$

# Objectives

- Objective 1: Find the velocity and diffusion coefficients in terms of position (x). So, $V(x)=?$ $D(x)=?$

- Objective 2: Overall, what do $V(x)$ and $D(x)$ depend on?