# Stability Analysis of Decentralized Supply Chains

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# Overview

- History and the bullwhip effect
- Mathematical and practical importance
- Current work
- Future work and suggestions

# What is the Bullwhip Effect?

- The bullwhip effect is what occurs when order fluctuations are greater upstream than downstream in the chain.
- Small fluctuations near initial conditions lead to large fluctuations long term.

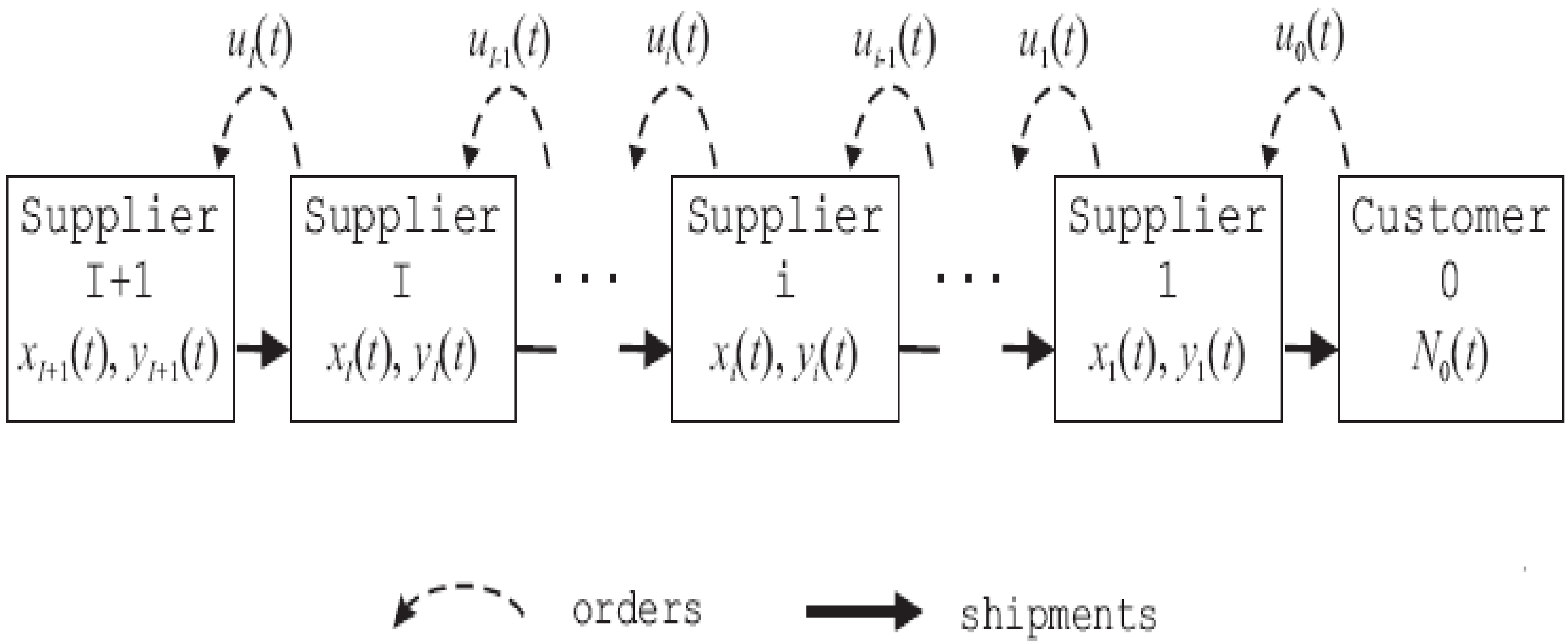
# The Importance of Analyzing the Bullwhip Effect

- The bullwhip effect inflates supply chain operating costs an estimated 12.5-25%
- The United States grocery industry could save an estimated $30 billion a year if the bullwhip effect is eliminated

# Questions Concerning the Bullwhip Effect

- Is it possible to dampen the bullwhip effect at the manufacturer level so that the manufacturer can produce a steady quantity efficiently?
- How many times can we eliminate the bullwhip effect in a supply chain?

# The Supply Chain




- Y Ouyang and C Daganzo Some properties of Decentralized Supply Chains figure 2

# Difference Equations

Inventory Position

$$x_i(t+1) = x_i(t) + u_i(t) - u_{i-1}(t) \quad \forall i = 1,2,\ldots$$

In-stock Inventory

$$y_i(t+1) = y_i(t) + u_i(t - l_i) - u_{i-1}(t) \quad \forall i = 1,2,\ldots$$

General Linear and Time Invariant (LTI) Ordering Policy

$$u_i(t) = \gamma_i + A_i(P)x_i(t) + B_i(P)y_i(t) + C_i(P)u_{i-1}(t-1) \quad \forall i = 1,2,\ldots$$

$$\gamma_i \in \mathbb{R}, \quad l_i \in \mathbb{N} \quad \forall i = 1,2,\ldots$$

$A_i(:)\ B_i(:)\ C_i(:)$ are polynomials with real coefficients

$P$ is a backward lag operator i.e $P^k x(t) = x(t-k)\ \forall k = 0,1,2,\ldots$

# Bullwhip Effect

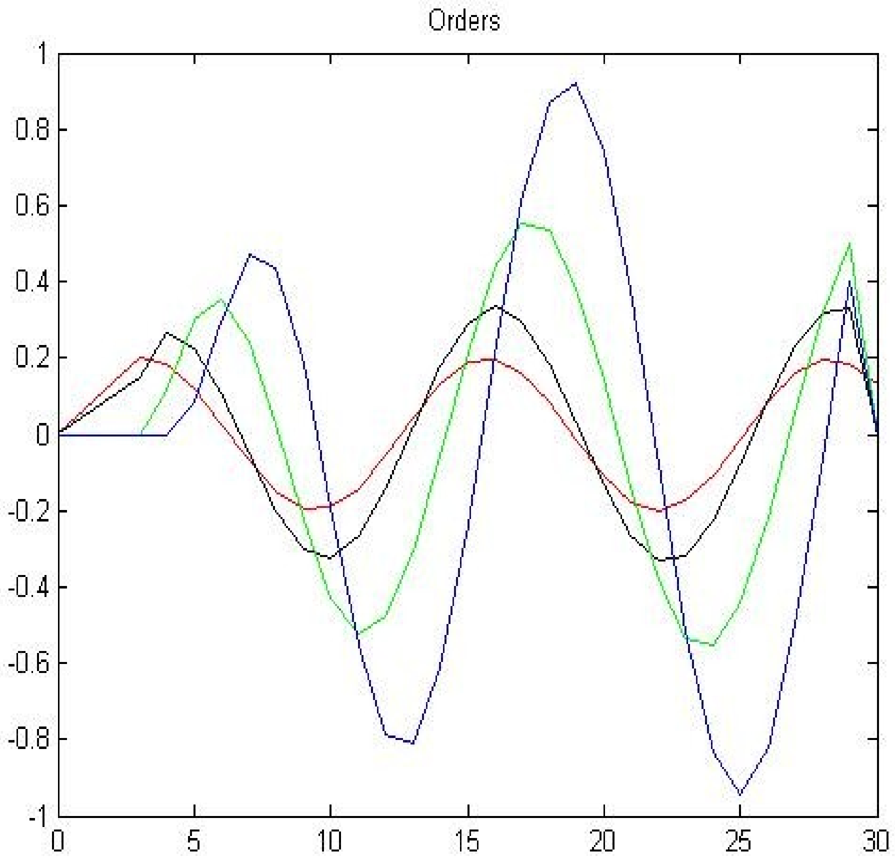
- **Definition:**

Supplier $\mathcal{J}+1$ ($\mathcal{J}>0$) in a supply chain experiences no bullwhip effect if the ratio of root mean square errors (RMSE) of the most upstream supplier and the customer demand is less than or equal to 1.

# Initial Conditions

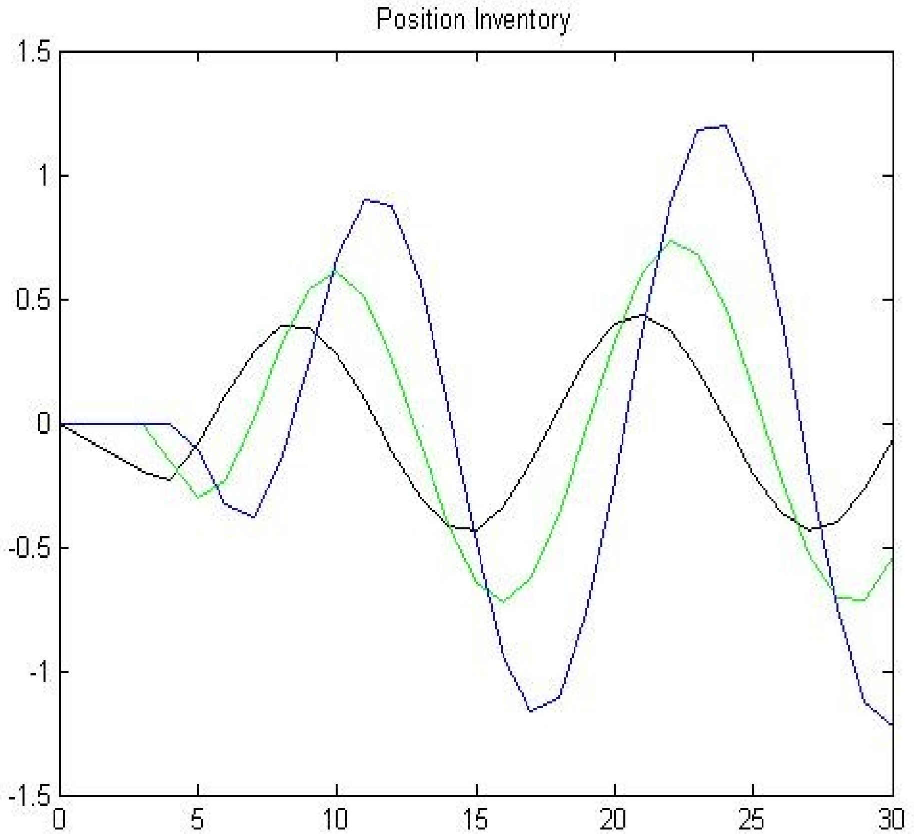
- The system starts at equilibrium
- Customer demand will be seasonal i.e. the demand will be sinusoidal but with different amplitudes and periods
- Lead time $l_i$=2

# Example of Bullwhip Effect in Orders Placed




- Customer
- Retailer
- Wholesaler
- Manufacturer
- $u_0(t)=0.2\sin(0.5t)$
- A(P)= -1/8
- B(P)= -1/8
- C(P)= $\frac{1+P}{2}$

# Inventory Position

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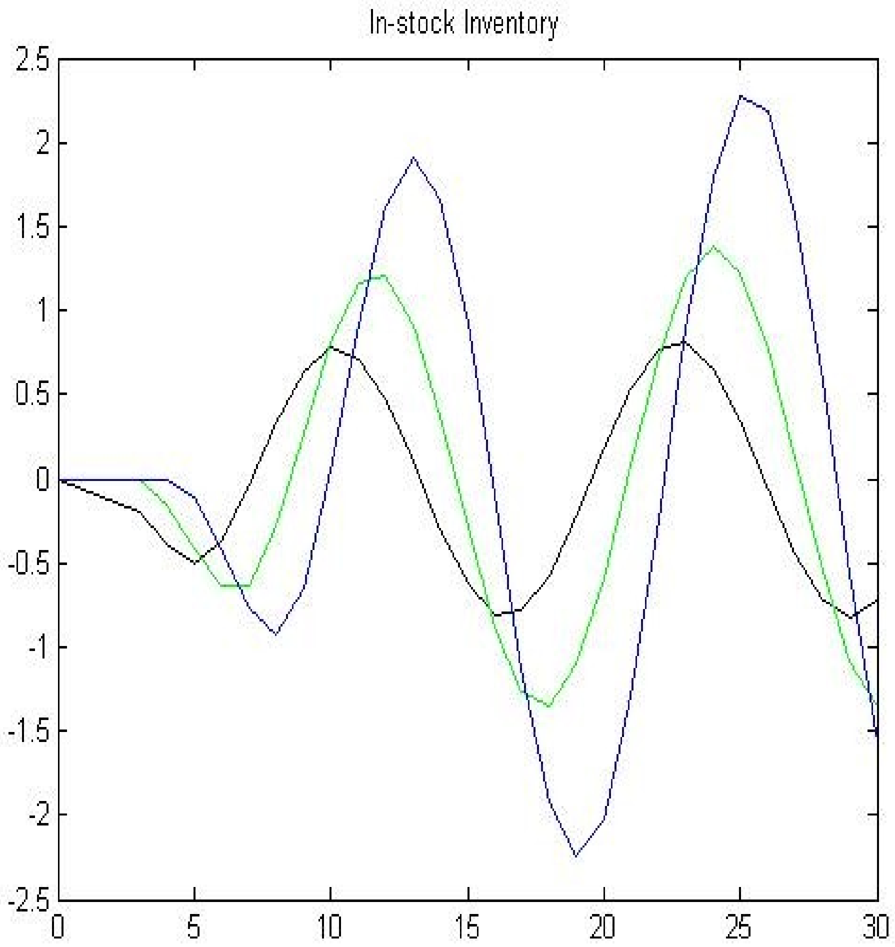
# In-stock Inventory




- Retailer
- Wholesaler
- Manufacturer

- $u_0(t)=0.2\sin(0.5t)$
- $A(P)= -1/8$
- $B(P)= -1/8$
- $C(P)= \frac{1+P}{2}$

# A Comparison of Two Policies

- Order-up-to policy

$$u_i(t) = -x_i(t) + u_{i-1}(t-1) + u_{i-1}(t-2) \quad \forall i, t$$

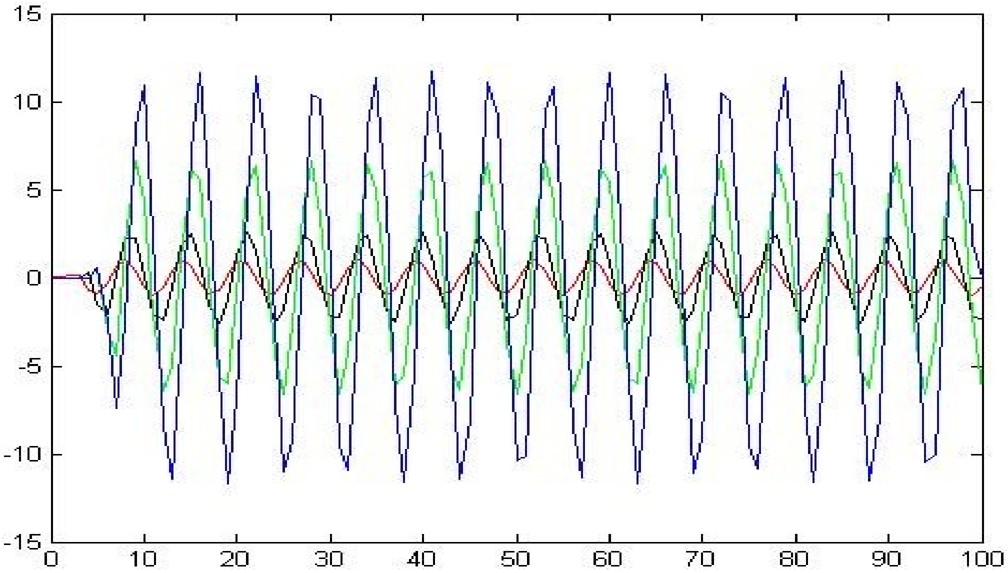
- Order based policy

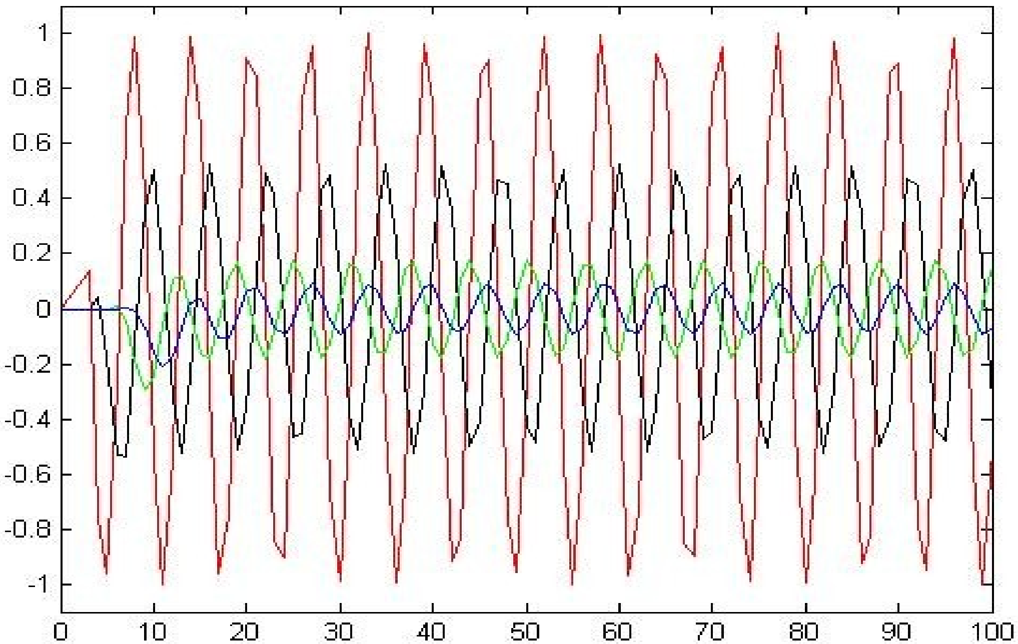
$$u_i(t) = 0.5\mathrm{u}_i(t) + 0.3\mathrm{u}_{i-1}(t-1) + 0.2\mathrm{u}_{i-1}(t-2) \quad \forall i, t$$

- *both functions will have the same $u_0(t)$ for all t

# A Comparison of Policies

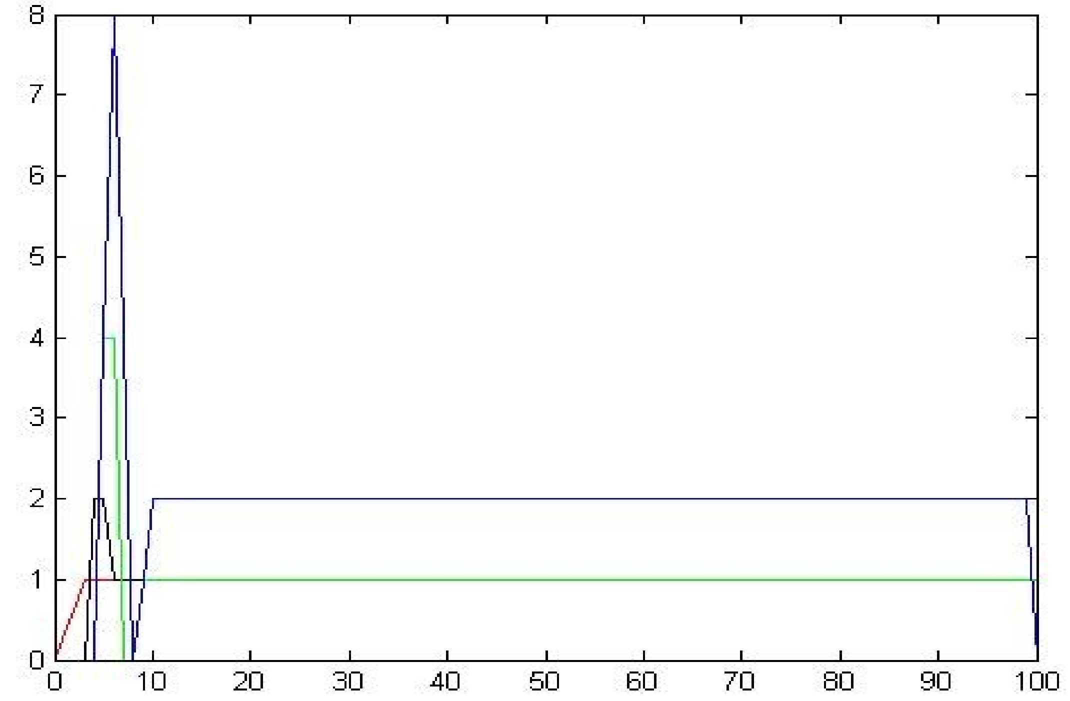
- Customer
- Retailer
- Wholesaler
- Manufacturer
- $u_0(t)=\sin(3t)$
- Order-up-to (top)
- Order based (bottom)

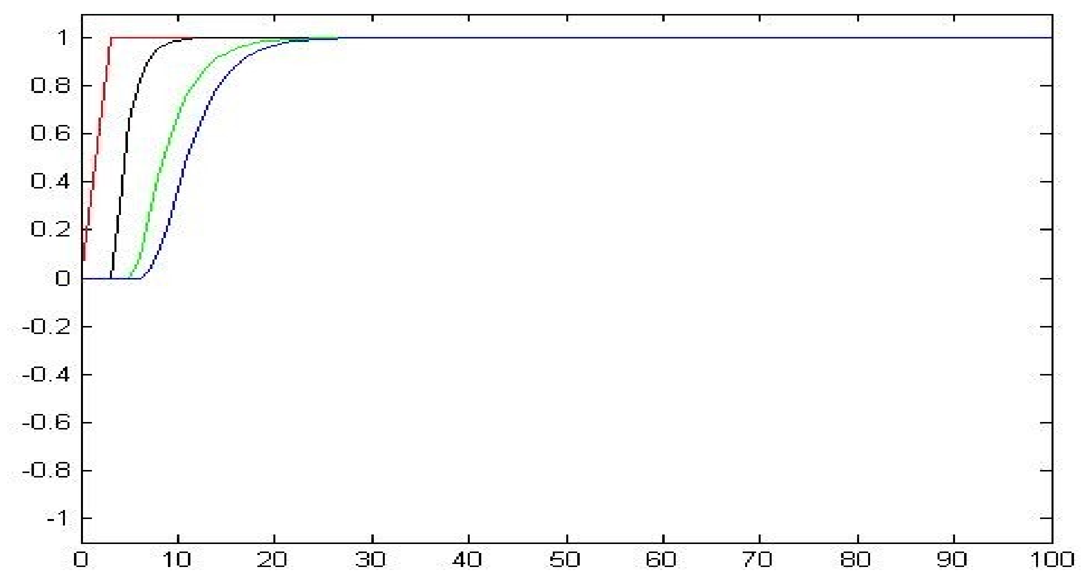


# A Comparison of Policies

- Customer
- Retailer
- Wholesaler
- Manufacturer
- $u_0(t)=1$
- Order-up-to (top)
- Order based (bottom)



# Z-Transformations

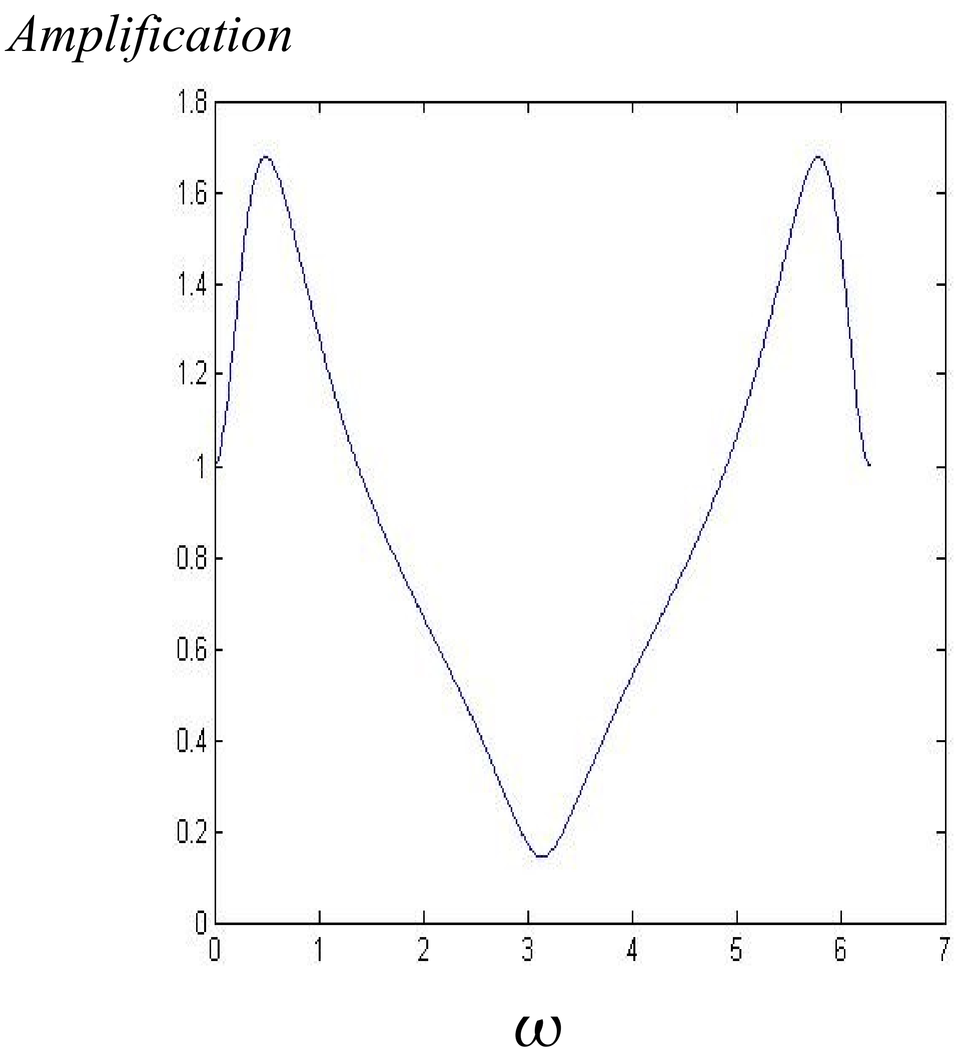
- Purpose: to look for policies that do not amplify the amplitude of any wave
- Reason: RMSE can be difficult to use to exhaust all arbitrary input order sequences especially if the chain is long and complex

# Transfer Function

$$T_{i-1,z} = \frac{z^{-1} C_i(z^{-1}) - (z-1)^{-1}[A_i(z^{-1}) + B_i(z^{-1})]}{1 - (z-1)^{-1}[A_i(z^{-1}) + z^{-l_i} B_i(z^{-1})]}$$

- Supplier I will not experience the bullwhip effect if
- $\sup_{\forall\, \omega[0,2\pi)} = |T_I(e^{j\omega})| \leq 1$

# Kanban Policy

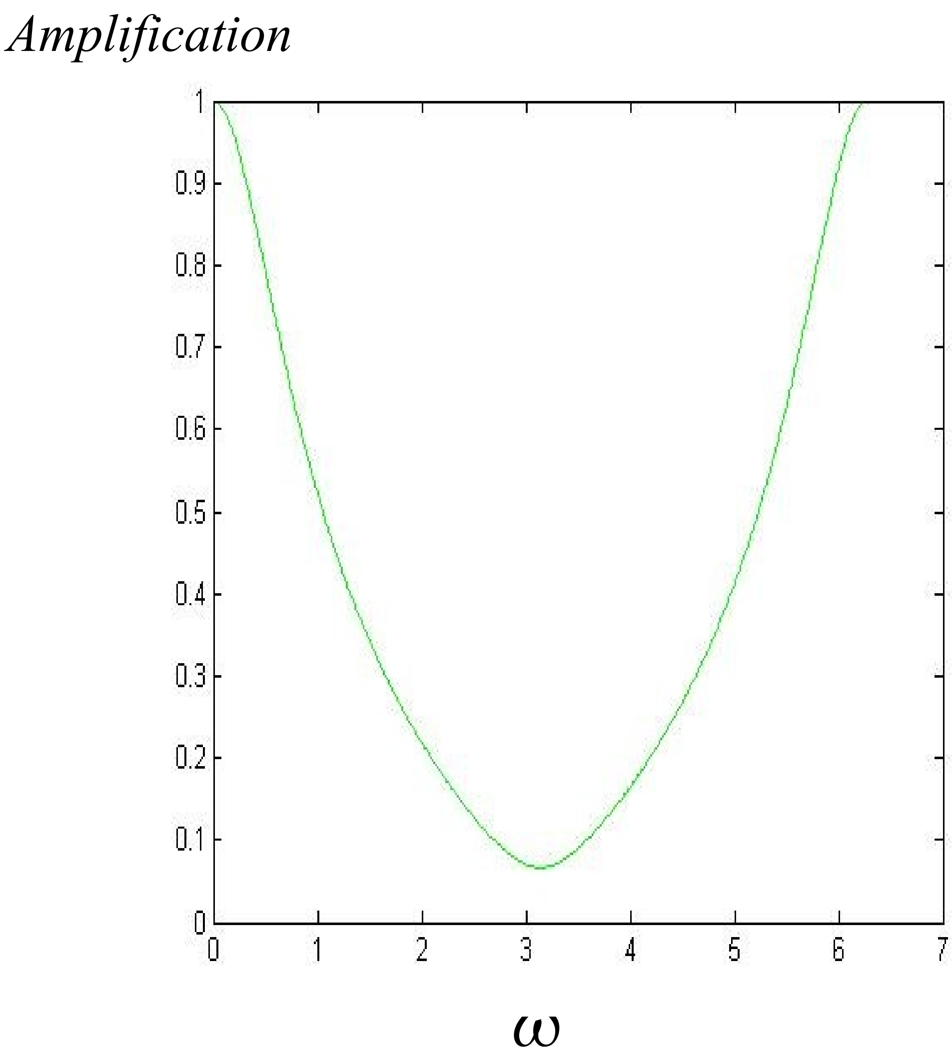



- A(P)=-1/8
- B(P)=-1/8
- C(P)=(1+P)/2

Z-Transformation

$$T = \frac{2z^2 - 1}{z^3}$$

# Order-based Policy




- A(P)=-1/2
- B(P)=0
- C(P)=-1/5

Z-Transformation

$$T=\frac{3z+2}{10z^2-5z}$$

# A Recap

- We have already seen that the order based policy is stable.
- Other policies such as order up to policy and kanban are also frequently used but are unstable
- Why use another policy?
- The answer is the in-stock inventory equilibrium

# A Comparison of Inventory

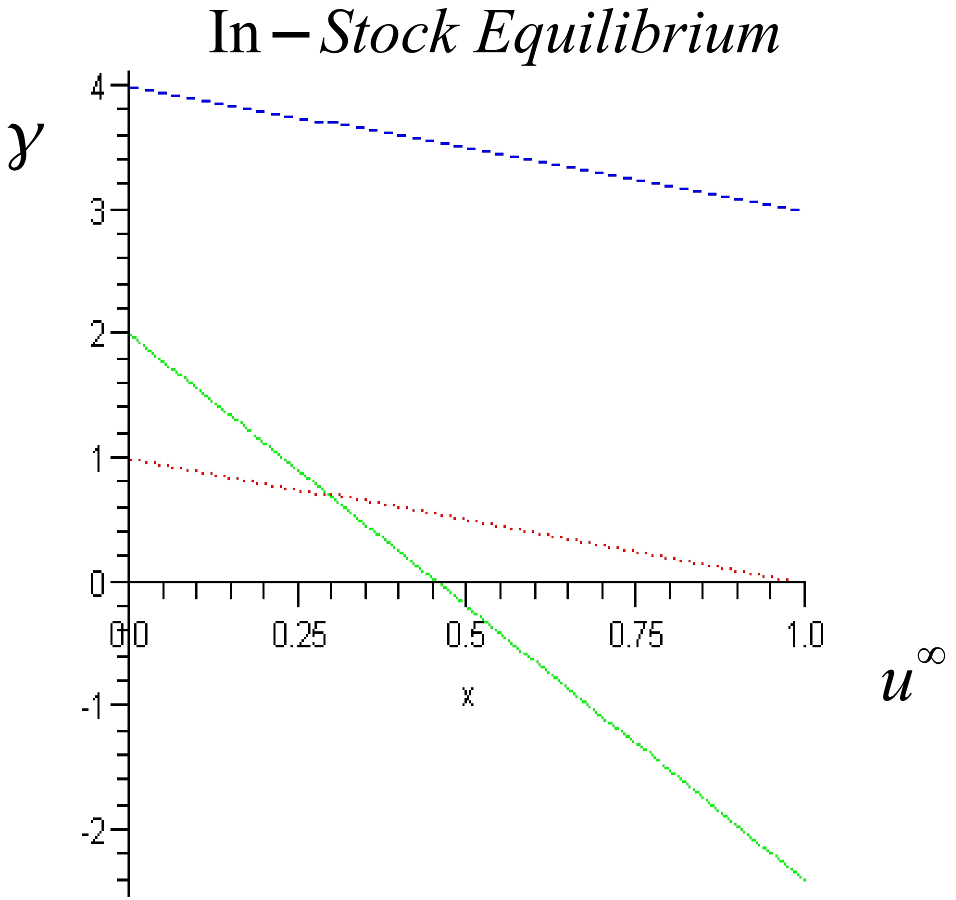
$$\text{In}-\textit{stock equilibrium} \quad y^{\infty}=\frac{\gamma}{A(1)+B(1)}+\frac{(1-A(1)l-C(1))u^{\infty}}{A(1)+B(1)}$$

$$\textit{Kanban} \quad y^{\infty}=4\gamma-u^{\infty}$$

$$\textit{Order}-\textit{based} \quad y^{\infty}=2\gamma-\frac{22}{5}u^{\infty}$$

$$\textit{Order up } \text{to} \quad y^{\infty}=\gamma-u^{\infty}$$

- Order-up to policy
- Order based policy
- Kanban policy



# Our Concerns

Is it possible to create a new ordering policy that is both stable and minimizes the in-stock inventory equilibrium?

How much information do we need to assume in order to be able to solve the problem?

# The Equation

$$L = \frac{\gamma}{A(1)+B(1)} + \frac{(1-A(1)l-C(1))u^{\infty}}{A(1)+B(1)}$$

$$+\lambda\left(\left|\frac{z^{-1}C(z^{-1})-(z-1)^{-1}[A(z^{-1})+B(z^{-1})]}{1-(z-1)^{-1}[A(z^{-1})+z^{-l}B(z^{-1})]}\right|-1\right)$$

To solve the system we make the following assumptions :

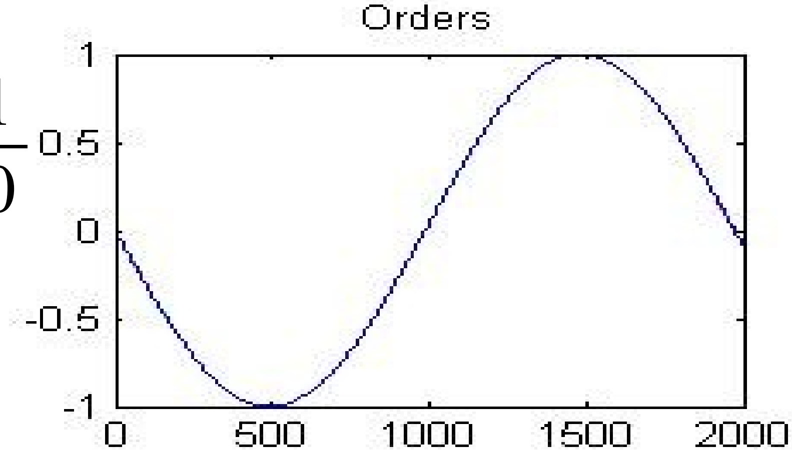
$$B(1)=\frac{1}{8} \qquad u^{\infty}=1 \qquad \gamma=1 \qquad l=2 \qquad z=e^{(i\omega)} \qquad \omega=\pi$$

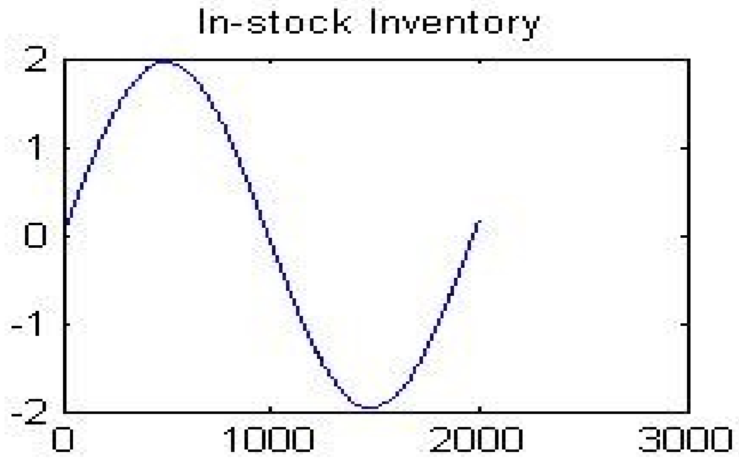
Solving the equation for the other variables yields

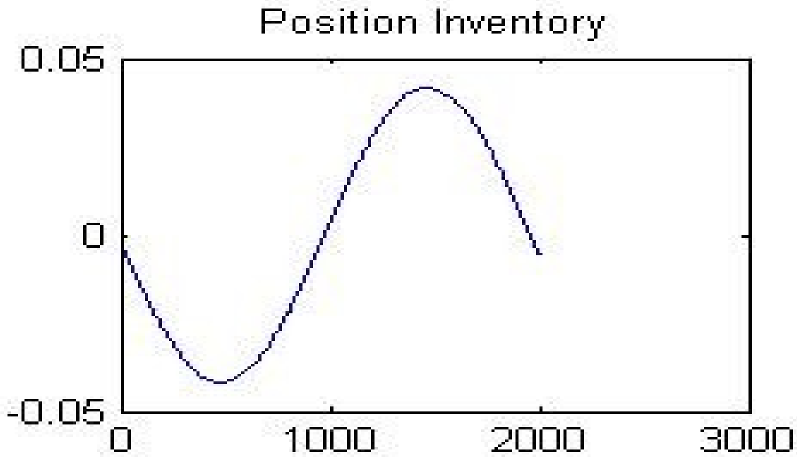
$$A(1)=\frac{-29}{40} \qquad C(1)=\frac{51}{40} \qquad \lambda=\frac{-7}{6}$$

# Solution for One Frequency

$$A(1)=\frac{-29}{40} \qquad B(1)=\frac{1}{8} \qquad C(1)=\frac{51}{40}$$



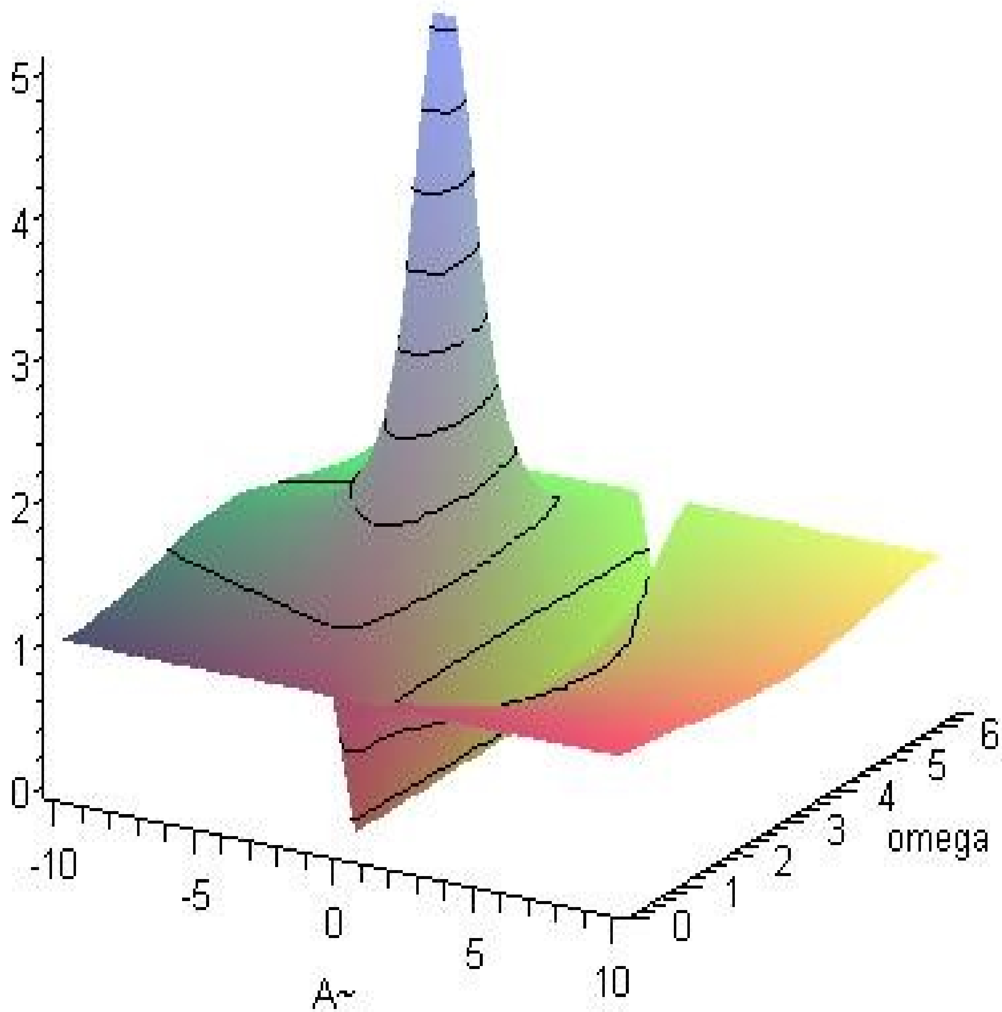



# Concerns

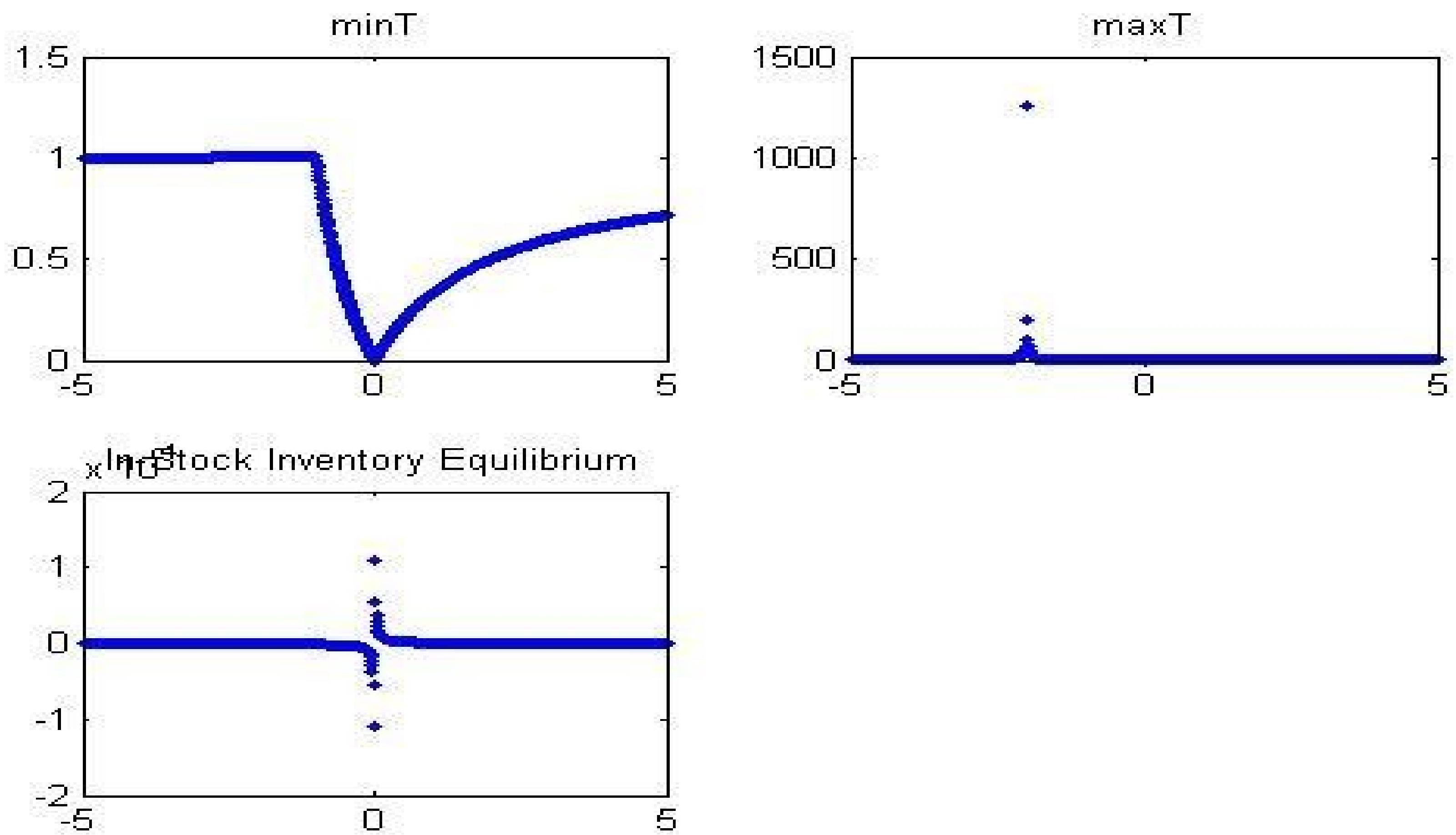
- Discretizing over the entire frequency is a very time consuming and lengthy process
- Some frequencies will not be handled by Maple and will need to be solved by hand
- Different values for the coefficients at each frequency

# In 1-Dimension




$$|T| = \left| \frac{A}{z - A - 1} \right|$$

# MATLAB Code


minT
1.5
1
0.5
0
-5
0
5
maxT
1500
1000
500
0
-5
0
5
In Stock Inventory Equilibrium
x 10^-3
2
1
0
-1
-2
-5
0
5