

# Stability Analysis of Decentralized Supply Chains

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# Overview

- History and the bullwhip effect
- Mathematical and practical importance
- Current work
- Future work and suggestions

# What is the Bullwhip Effect?

- The bullwhip effect is what occurs when order fluctuations are greater upstream than downstream in the chain.
- Small fluctuations near initial conditions lead to large fluctuations long term.

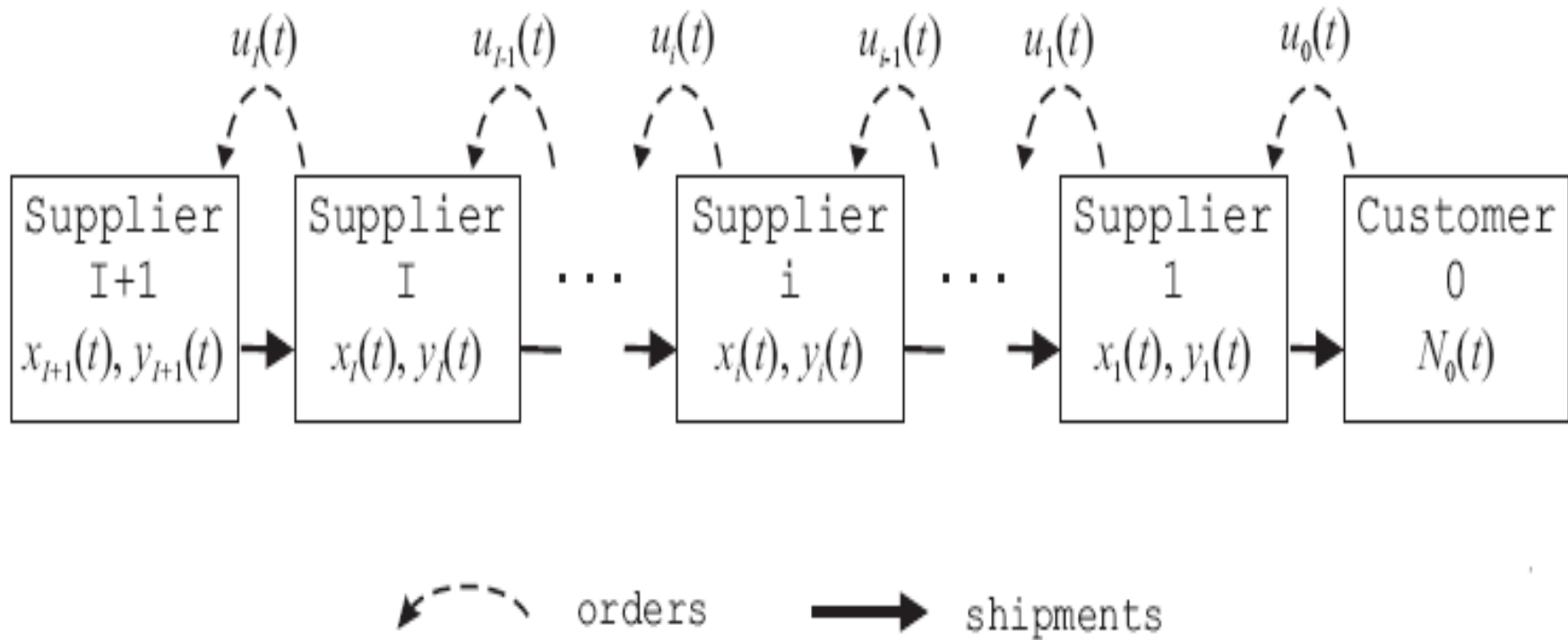
# The Importance of Analyzing the Bullwhip Effect

- The bullwhip effect inflates supply chain operating costs an estimated 12.5-25%
- The United States grocery industry could save an estimated \$30 billion a year if the bullwhip effect is eliminated

# Questions Concerning the Bullwhip Effect

- Is it possible to dampen the bullwhip effect at the manufacturer level so that the manufacturer can produce a steady quantity efficiently?
- How many times can we eliminate the bullwhip effect in a supply chain?

# The Supply Chain



- Y Ouyang and C Daganzo Some properties of Decentralized Supply Chains figure 2

# Difference Equations

Inventory Position

$$x_i(t+1) = x_i(t) + u_i(t) - u_{i-1}(t) \quad \forall i=1,2,\dots$$

In-stock Inventory

$$y_i(t+1) = y_i(t) + u_i(t - l_i) - u_{i-1}(t) \quad \forall i=1,2,\dots$$

General Linear and Time Invariant (LTI) Ordering Policy

$$u_i(t) = \gamma_i + A_i(P) x_i(t) + B_i(P) y_i(t) + C_i(P) u_{i-1}(t-1) \quad \forall i=1,2,\dots$$

$$\gamma_i \in \mathbb{R}, \quad l_i \in \mathbb{N} \quad \forall i=1,2,\dots$$

$A_i(\cdot)$   $B_i(\cdot)$   $C_i(\cdot)$  are polynomials with real coefficients

$P$  is a backward lag operator i.e.  $P^k x(t) = x(t-k) \forall k=0,1,2,\dots$

# Bullwhip Effect

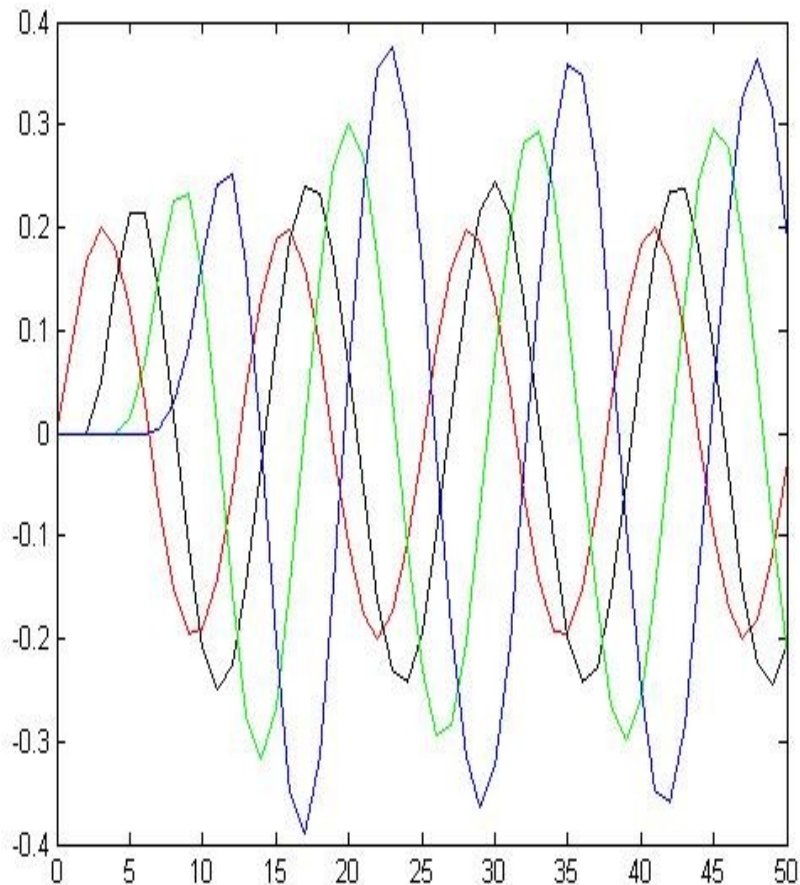
- **Definition:**

Supplier  $I+1$  ( $I > 0$ ) in a supply chain experiences no bullwhip effect if the ratio of root mean square errors (RMSE) of the most upstream supplier and the customer demand is less than or equal to 1.

# Initial Conditions

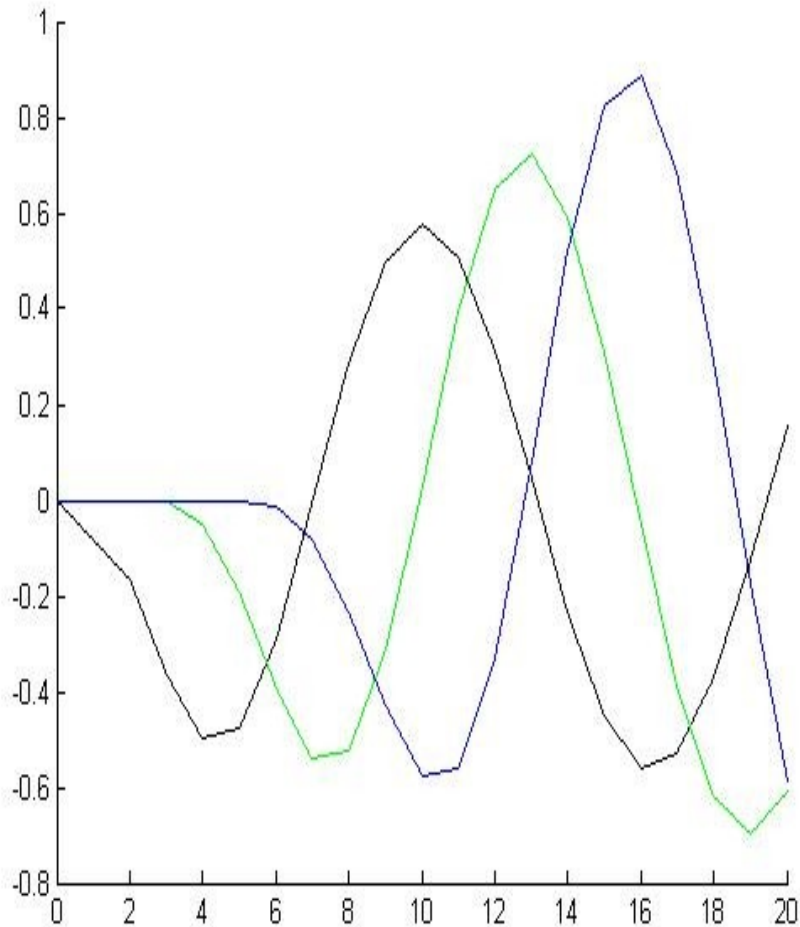
- The system starts at equilibrium
- Customer demand will be seasonal i.e. the demand will be sinusoidal
- Lead time  $l_i = 1$

# Example of Bullwhip Effect in Orders Placed



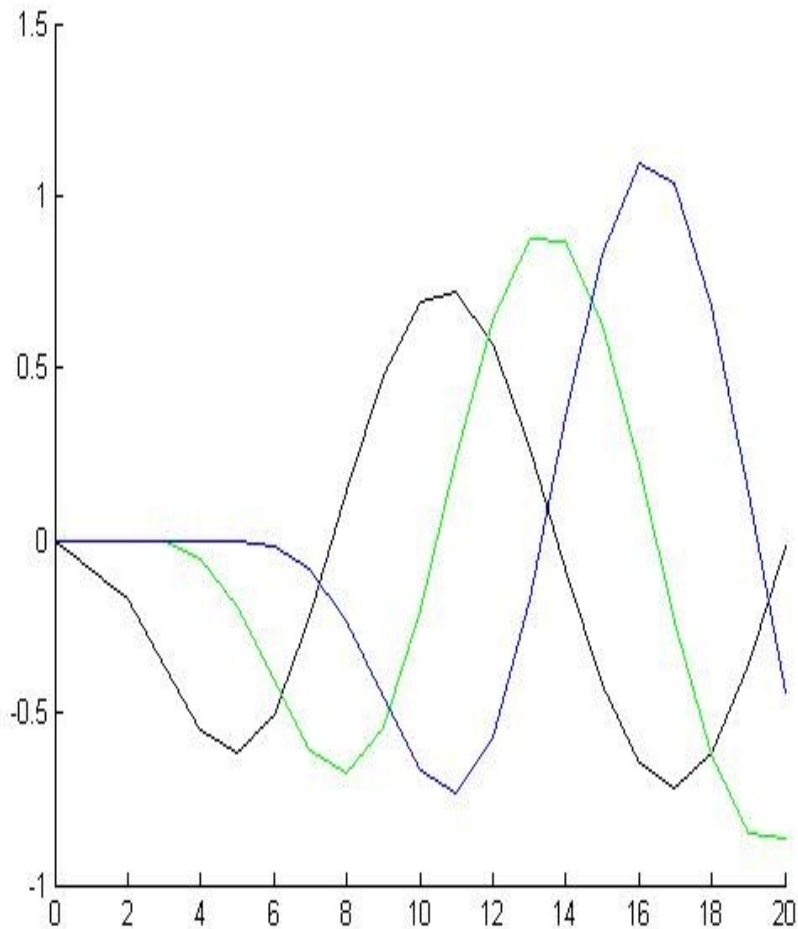
- Customer
- Retailer
- Wholesaler
- Manufacturer
- $u_0(t) = 0.2\sin(0.5t)$
- $A(P) = -0.5$
- $B(P) = 0$
- $C(P) = -0.2$

# Inventory Position



- Retailer
- Wholesaler
- Manufacturer
- $u_0(t) = 0.2 \sin(0.5t)$
- $A(P) = -0.5$
- $B(P) = 0$
- $C(P) = -0.2$

# In-stock Inventory



- Retailer
  - Wholesaler
  - Manufacturer
- 
- $u_0(t)=0.2\sin(0.5t)$
  - $A(P)= -0.5$
  - $B(P)= 0$
  - $C(P)= -0.2$

# A Comparison of Two Policies

- Order-up-to policy

$$u_i(t) = -x_i(t) + u_{i-1}(t-1) + u_{i-1}(t-2) \quad \forall i, t$$

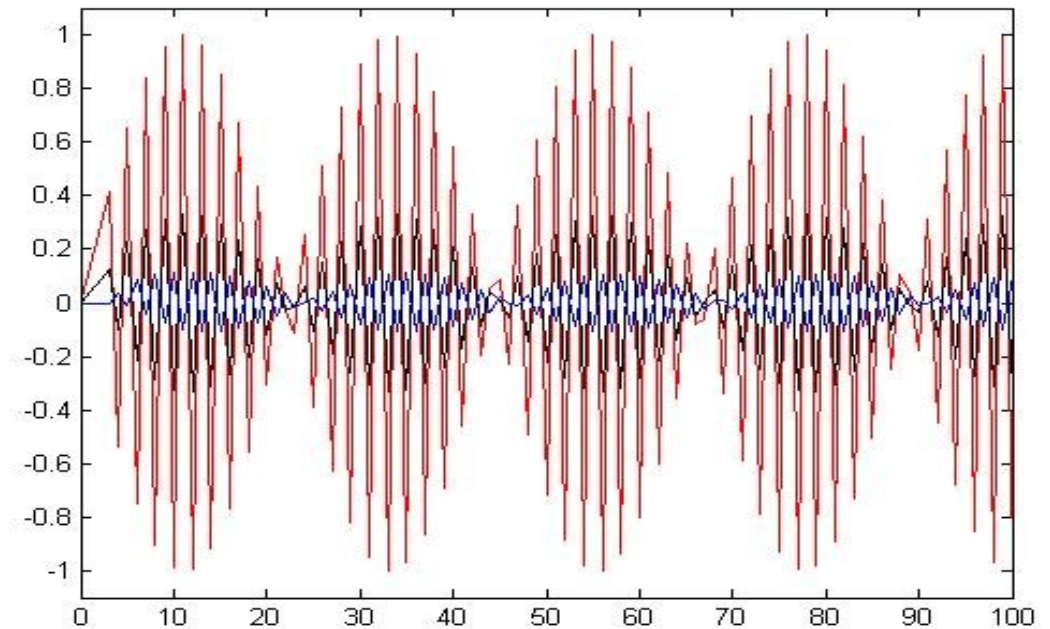
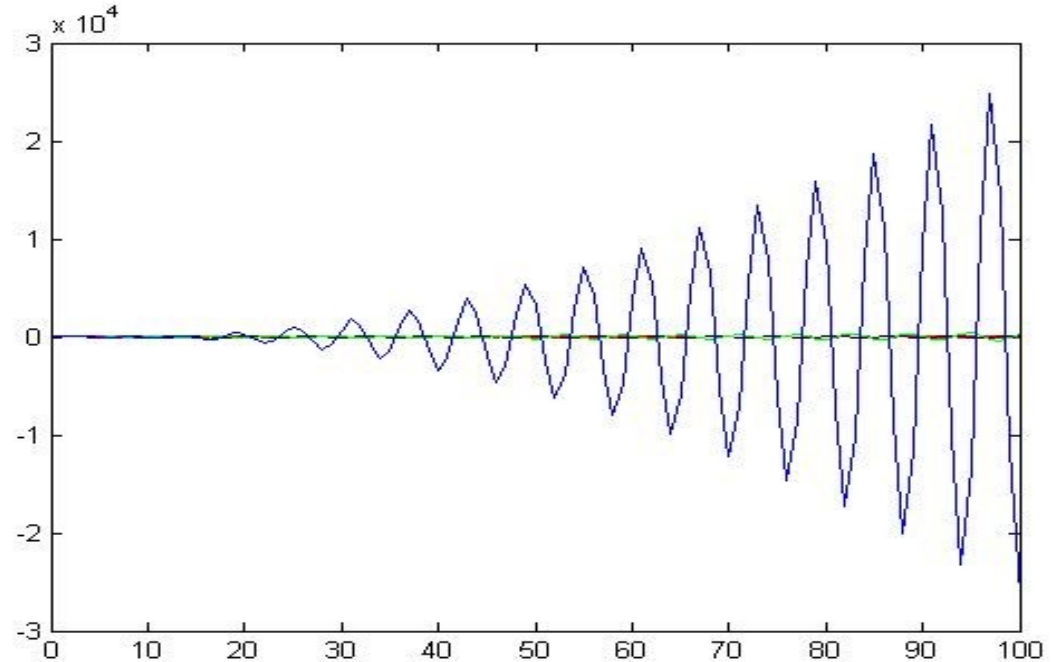
- Order based policy

$$u_i(t) = 0.5u_i(t-1) + 0.3u_{i-1}(t-1) + 0.2u_{i-1}(t-2) \quad \forall i, t$$

- \*both functions will have the same  $u_0(t)$  for all  $t$

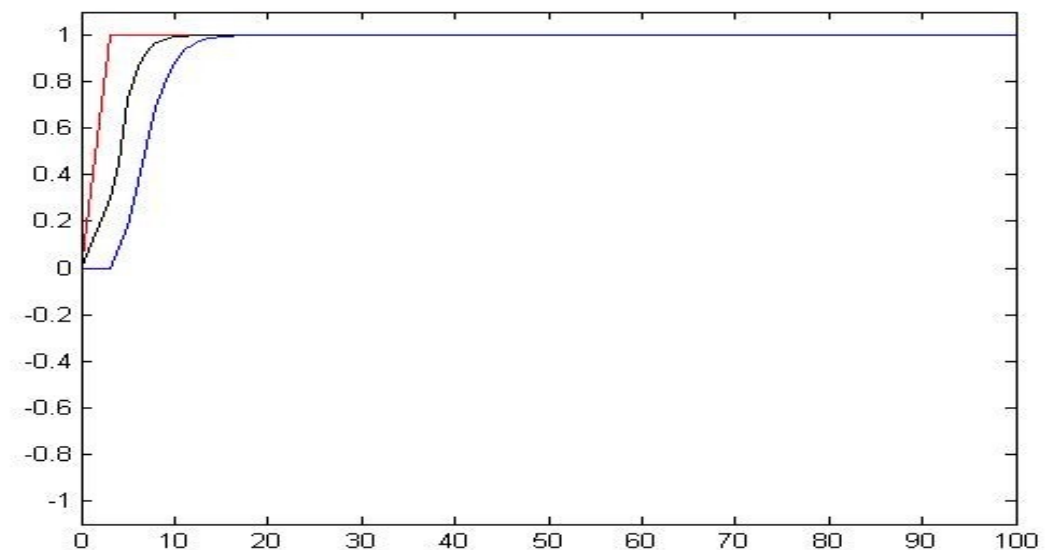
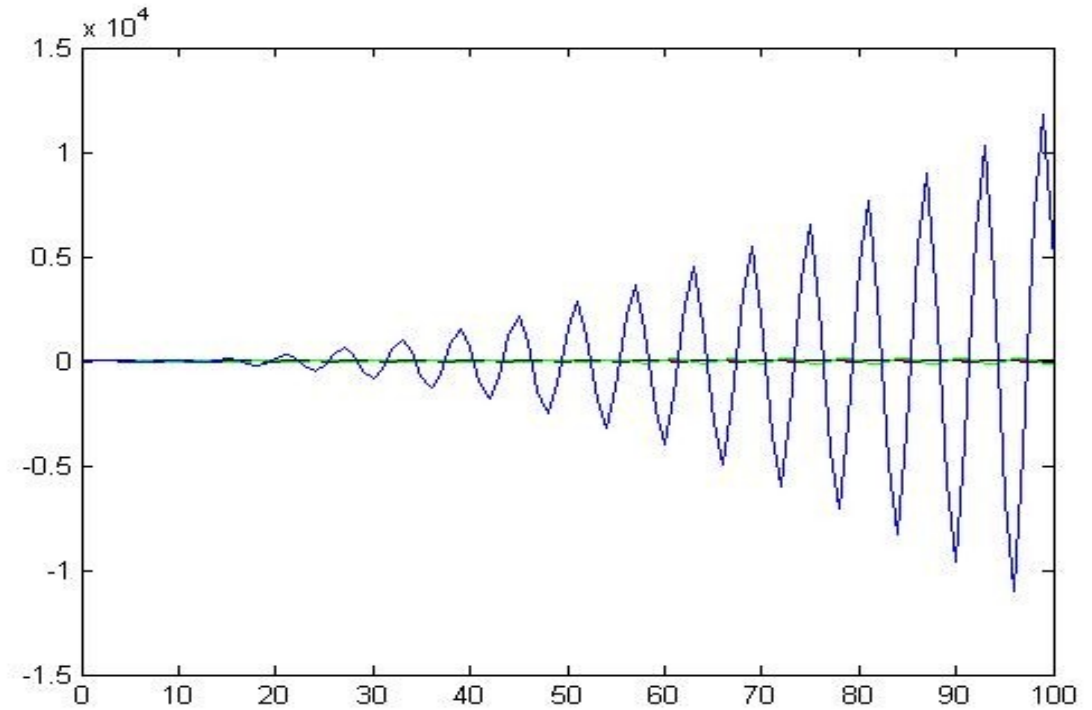
# A Comparison of Policies

- Customer
- Retailer
- Wholesaler
- Manufacturer
- $u_0(t) = \sin(3t)$
- Order-up-to (top)
- Order based (bottom)



# A Comparison of Policies

- Customer
- Retailer
- Wholesaler
- Manufacturer
- $u_0(t)=1$
- Order-up-to (top)
- Order based (bottom)



# Z-Transformations

- Purpose: to look for policies that do not amplify the amplitude of any wave
- Reason: RMSE can be difficult to use to exhaust all arbitrary input order sequences especially if the chain is long and complex

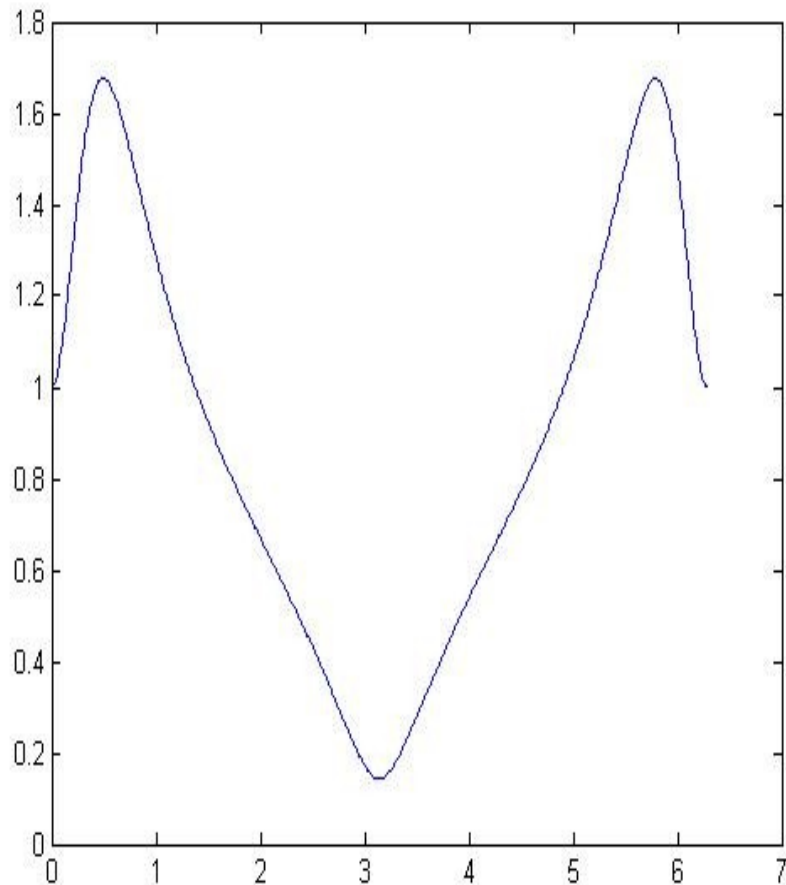
# Transfer Function

$$T_{i-1,z} = \frac{z^{-1}C_i(z^{-1}) - (z-1)^{-1}[A_i(z^{-1}) + B_i(z^{-1})]}{1 - (z-1)^{-1}[A_i(z^{-1}) + z^{-l_i}B_i(z^{-1})]}$$

- Supplier I will not experience the bullwhip effect if
- $\sup_{\forall \omega \in [0, 2\pi)} |T_i(e^{j\omega})| \leq 1$

# Kanban Policy

- $A(P) = -1/8$
- $B(P) = -1/8$
- $C(P) = (1+P)/2$

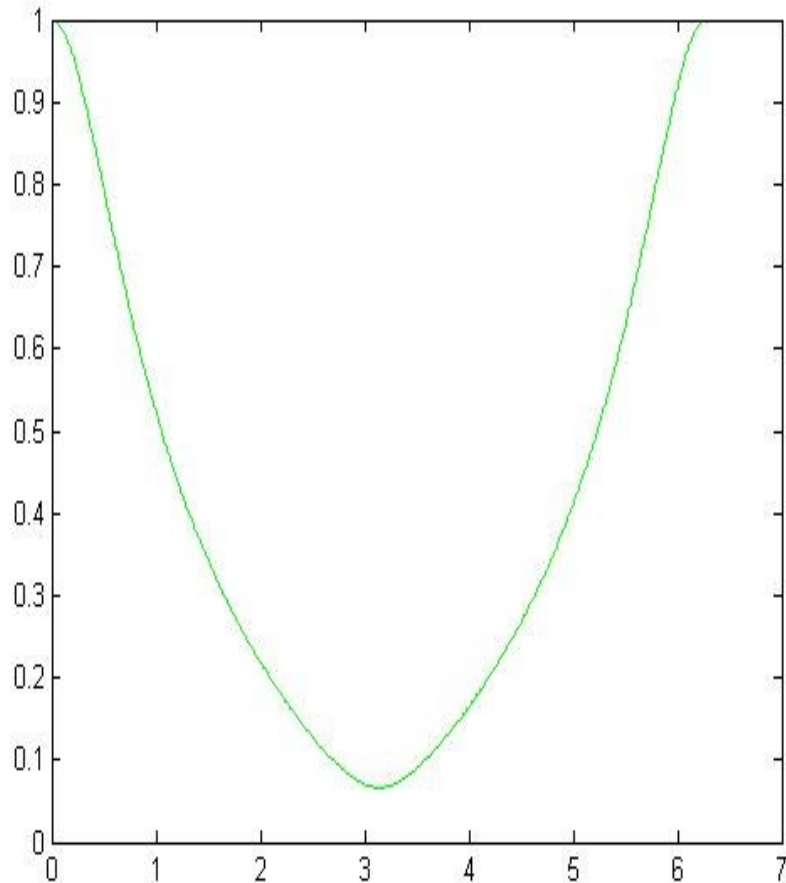


Z-Transformation

$$T = \frac{2z^2 - 1}{z^3}$$

# Order-based Policy

- $A(P)=-1/2$
- $B(P)=0$
- $C(P)=-1/5$



Z-Transformation

$$T = \frac{3z + 2}{10z^2 - 5z}$$

# Future Work

- Look at inhomogeneous chains
- Split demand at  $I-1$  and see if we can dampen the bullwhip effect by using two different policies
- \*Stochastically-Operated Chains