Thoughts towards PDE

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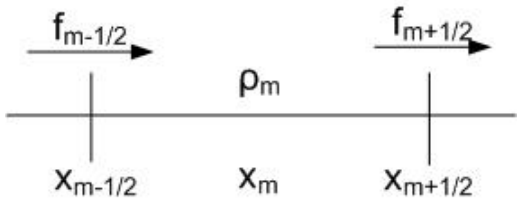
The setup



$$\rho_{m,t+1} = \rho_{m,t} + \frac{\Delta t}{\Delta m}\left(f_{m-\frac{1}{2},t} - f_{m+\frac{1}{2},t}\right) \qquad (1)$$

Cases

1. No blocking of m ($\rho_{m,t+1} < \rho_{max}$)

$$f_{m-\frac{1}{2},t} = Q_{m-1,max}[1 - e^{-b_{m-1}\rho_{m-1}(t)}] \tag{2}$$

2. Blocking of m ($\rho_{m,t} = \rho_{max}$)

$$f_{m-\frac{1}{2},t} = f_{m+\frac{1}{2},t} \tag{3}$$

3. No Blocking yet, but blocking of m in next timestep

$$f_{m-\frac{1}{2},t} = \frac{\Delta m}{\Delta t}(\rho_{max} - \rho_{m,t}) + f_{m+\frac{1}{2},t} \tag{4}$$

$$\rho_{m,t} < \rho_{max}$$

$$\rho_{m,t} + \frac{\Delta t}{\Delta m}\left(f_{m-\frac{1}{2},t} - f_{m+\frac{1}{2},t}\right) = \rho_{max}$$

Combining Equation 2 and 4 gives:

$$f_{m-\frac{1}{2},t} = min[Q_{m,t}, \frac{\triangle m}{\triangle t}(\rho_{max} - \rho_{m,t}) + f_{m+\frac{1}{2},t}] \tag{5}$$

However, since:

$$f_{m+\frac{1}{2},t} = Q^2_{m,max}[1 - e^{-b^2_m \rho_m(t)}] \tag{6}$$

combining these Equations and setting $Q_{m,t} = Q^1_{m,t}$ gives :

$$f_{m-\frac{1}{2},t} = min[Q^1_{m,t}, \frac{\triangle m}{\triangle t}(\rho_{max} - \rho_{m,t}) + Q^2_{m,t}] \tag{7}$$

PDE

Proposed continuous model:

$$\partial_t \rho + \partial_x f(\rho) = 0 \tag{8}$$

with

$$f(\rho) = \min\left(Q^1(\rho),\ C(\rho_{max} - \rho) + Q^2(\rho)\right) \tag{9}$$

where

$$C = \frac{f(\rho) - f(\rho_{max})}{(\rho - \rho_{max})} \tag{10}$$