

Simulating blocking behavior of manufactory lines

# Backwave: From DEM to PDE

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► DEM

Introduction

Example

Deterministic case

Deterministic case

Stochastic case: ...

Wavespeed equation

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Calculation of flux: ...

Questions?

Heuristics for ...

Simulation Results

# DEM

## Results of the $\chi$ simulation

# Introduction



**G** : Generator

**B** : Buffer

$B_{max}$  : Maximum workstation capacity (buffer + machine)

**M** : Machine

**E** : Exit

$t_a, c_a^2$  : inter-arrival time and variance

$t_{eL}, t_{eH}, c_e^2$  : low and high process times and variance

where :  $t_{eL} < t_a < t_{eH}$

DEM

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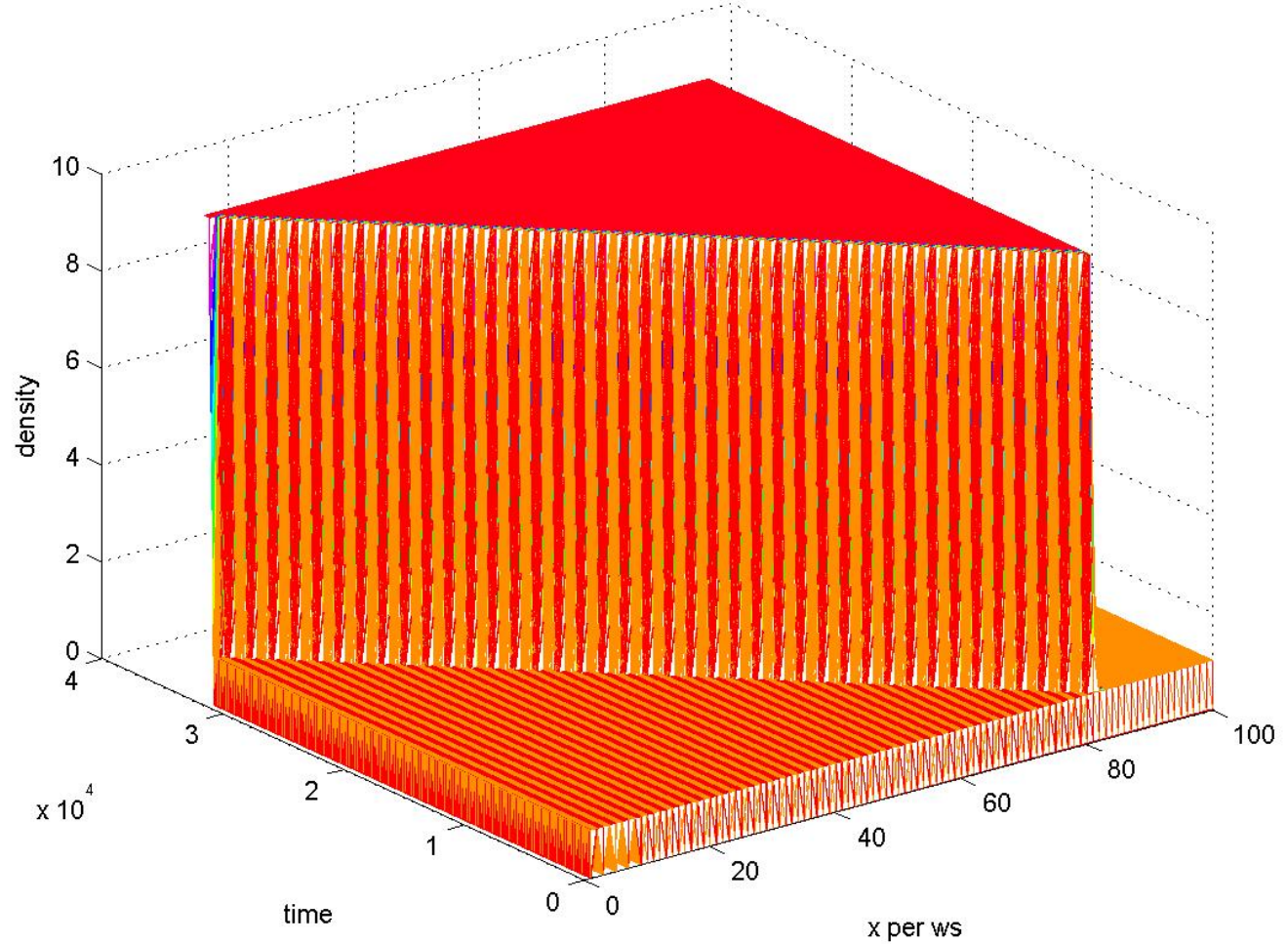
Simulation Results

# Example

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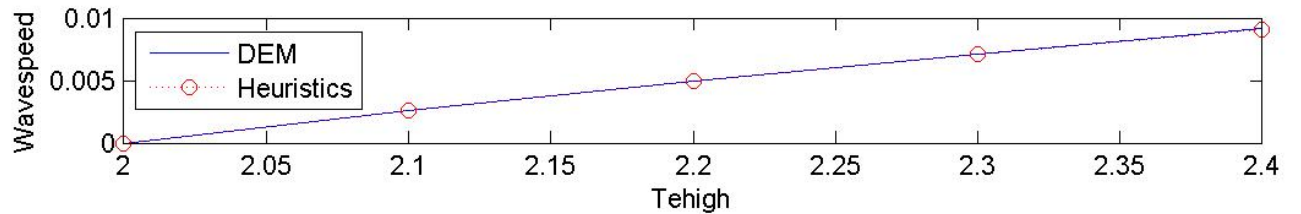
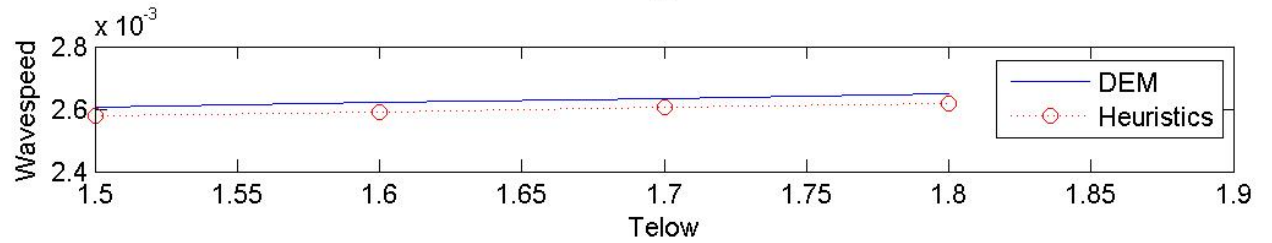
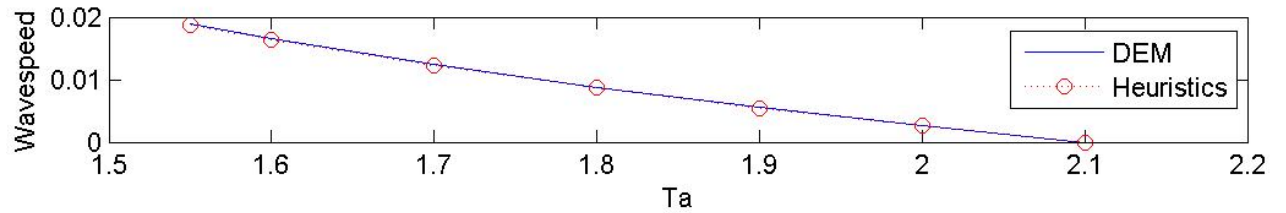
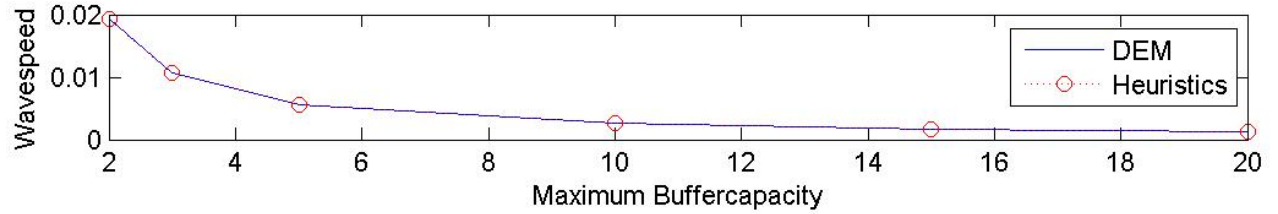
# Deterministic case

Density in the deterministic case



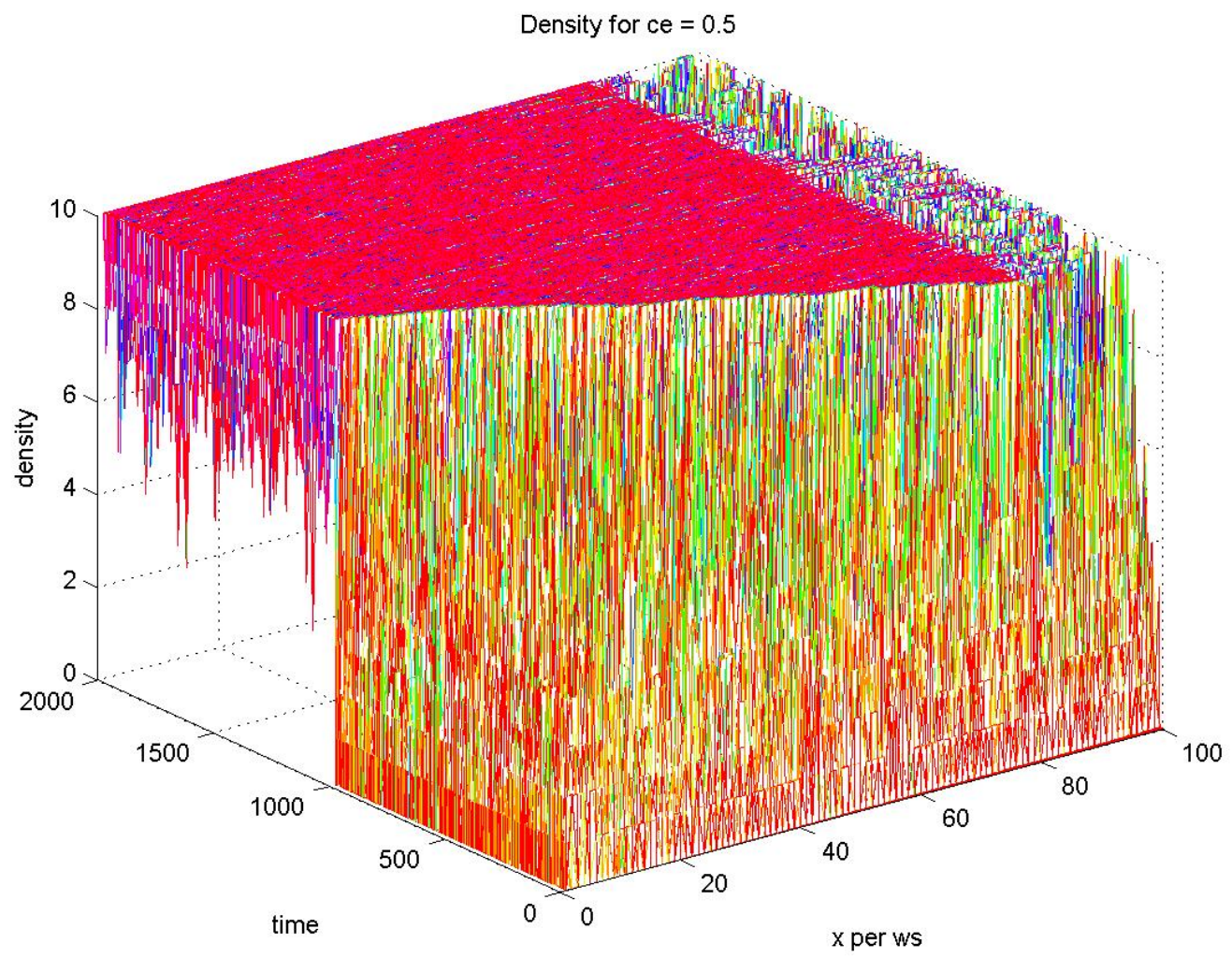
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# Stochastic case: Variance in process time



# Wavespeed equation

$$v_{wave} = \frac{\sum_{m=1, t+\Delta t}^{m=M} WIP_{(m, t+\Delta t)} - \sum_{m=1, t}^{m=M} WIP_{(m, t)}}{\Delta t} \quad (1)$$

where:

$M$  : number of workstations

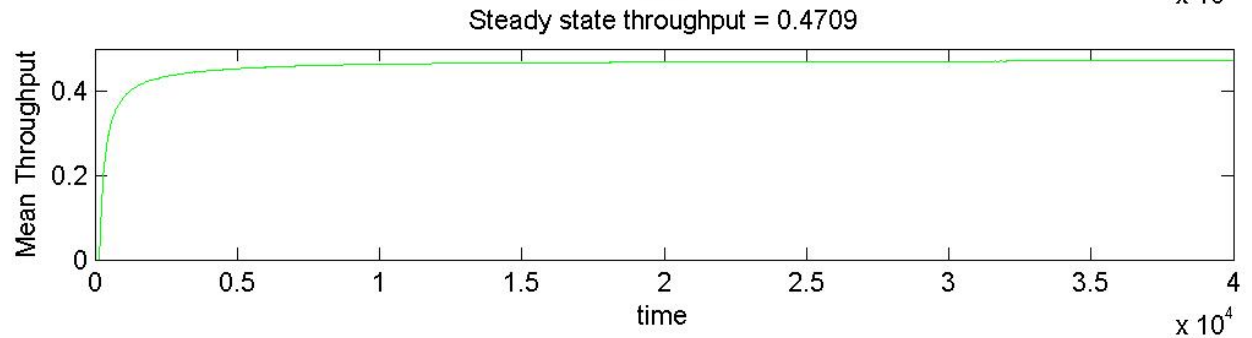
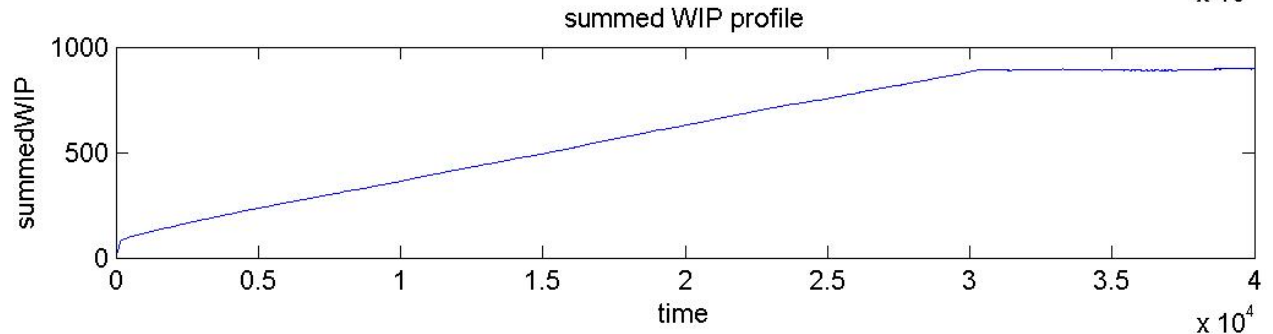
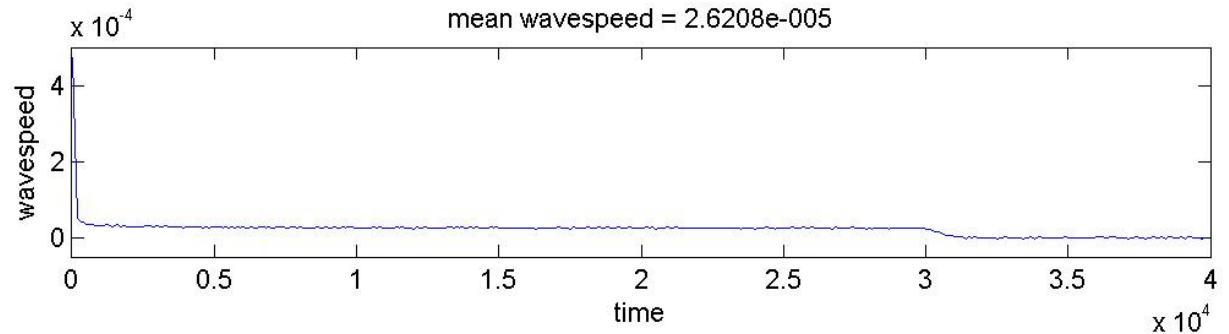
$\Delta t$  : time interval

$WIP_{(m, t)}$  : Work in Progress at workstation  $m$  at time  $t$

$v_{wave}$  : wavespeed in interval  $\Delta t$

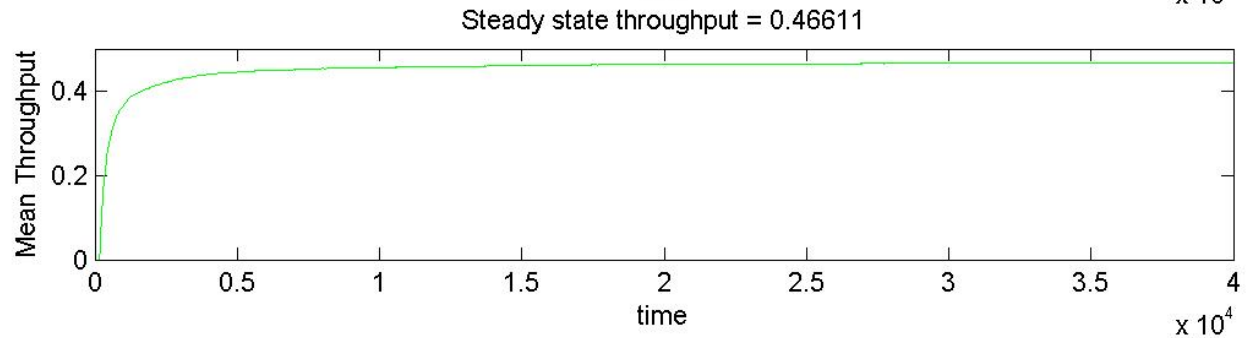
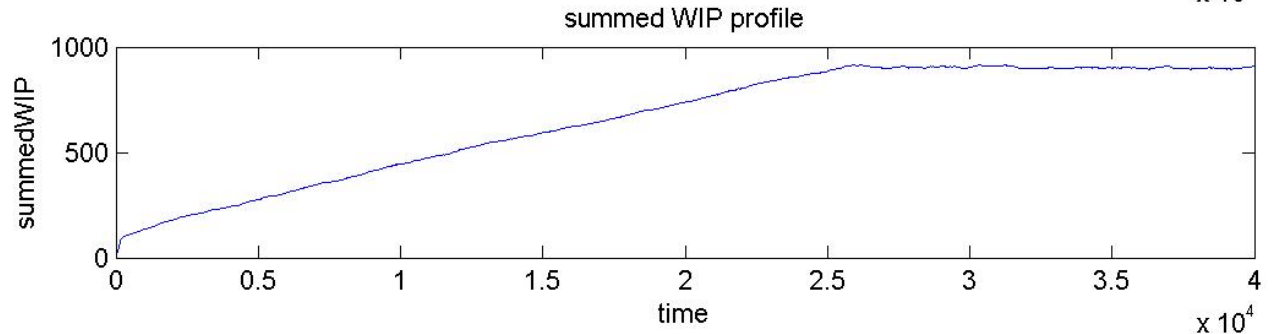
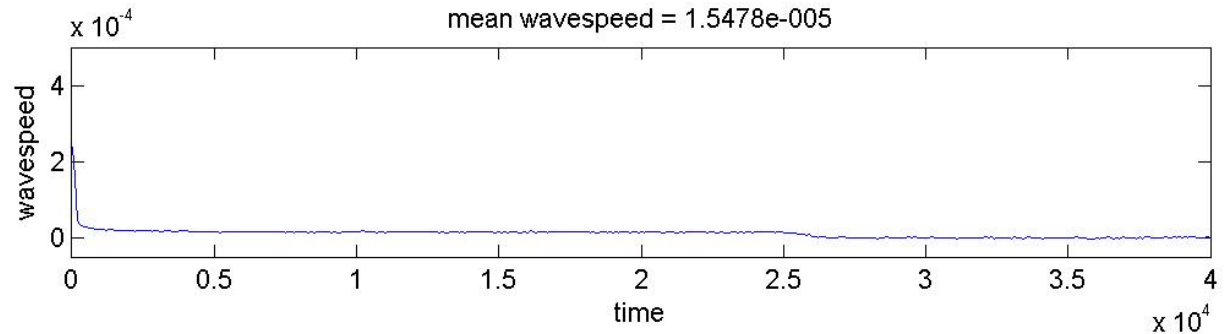
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# Stochastic Results



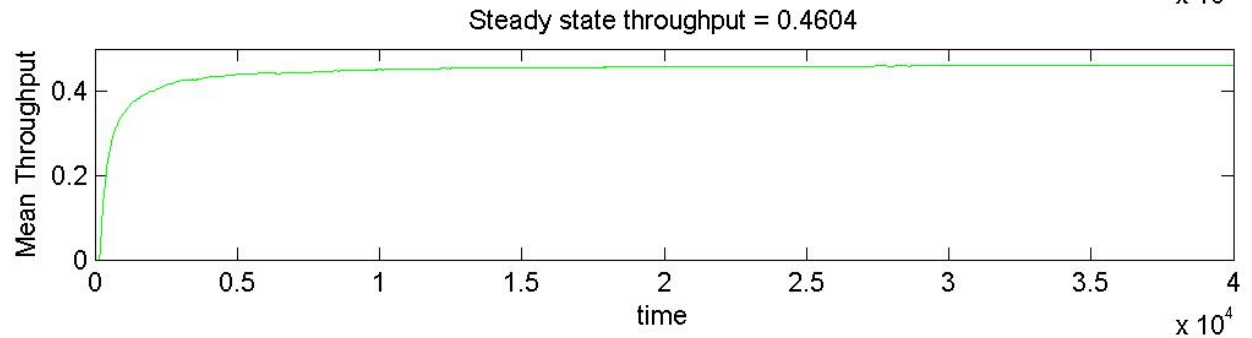
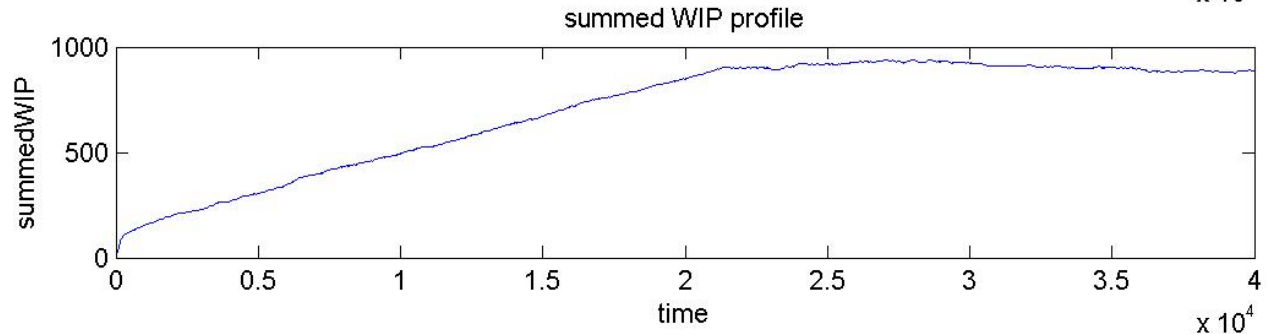
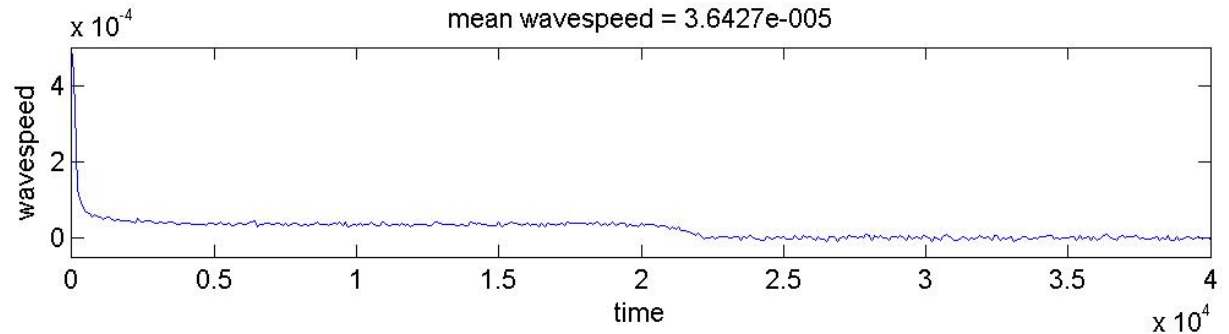
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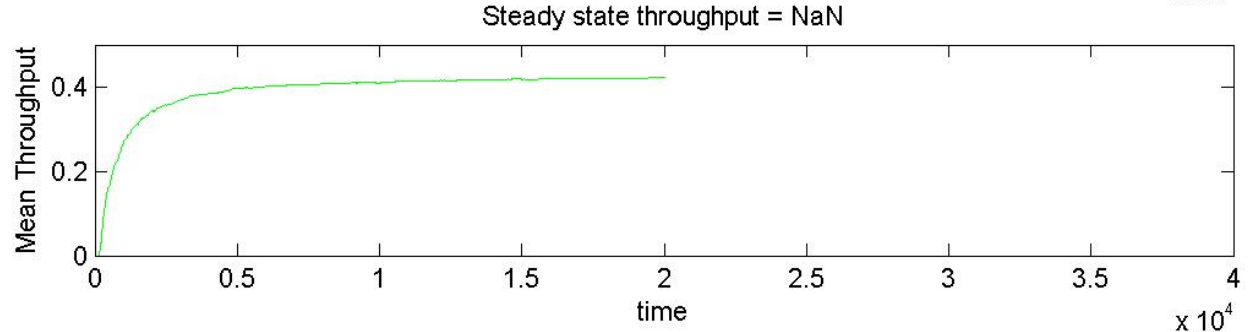
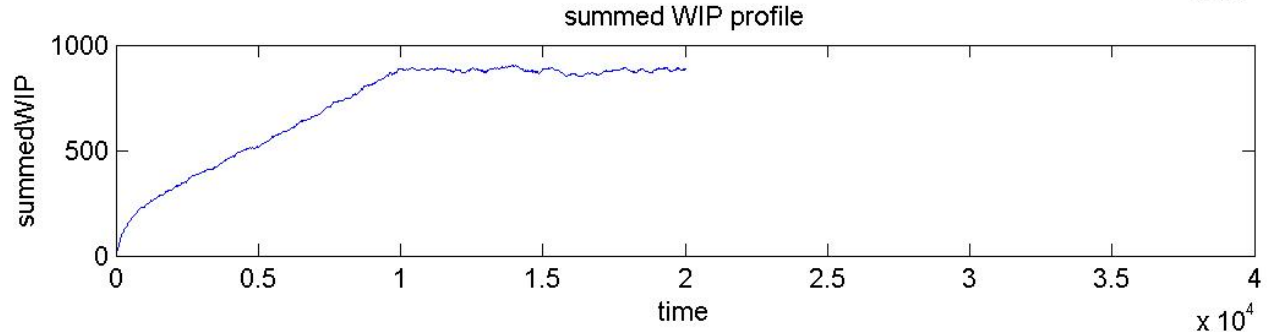
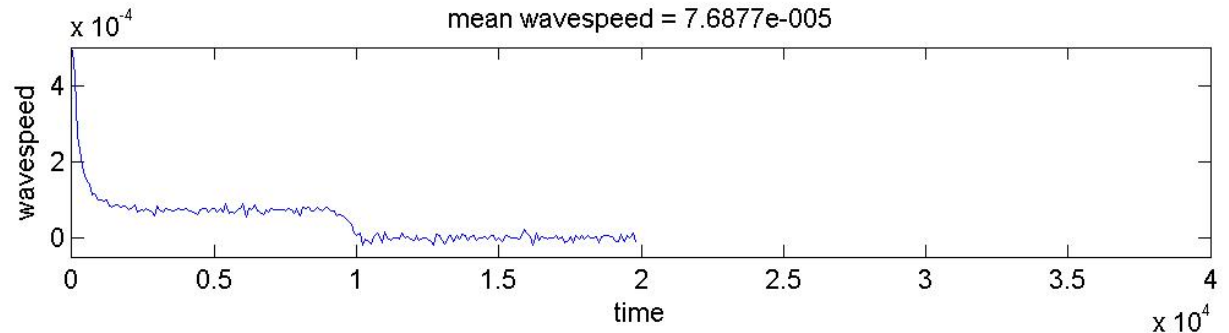
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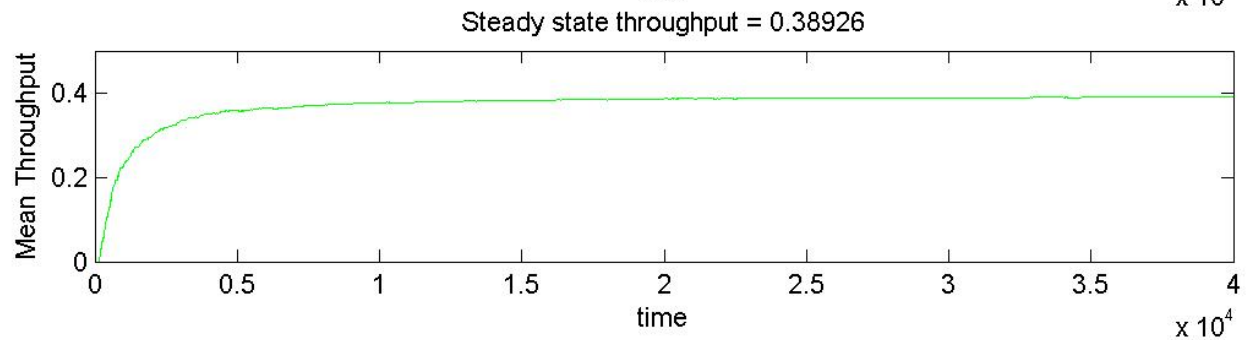
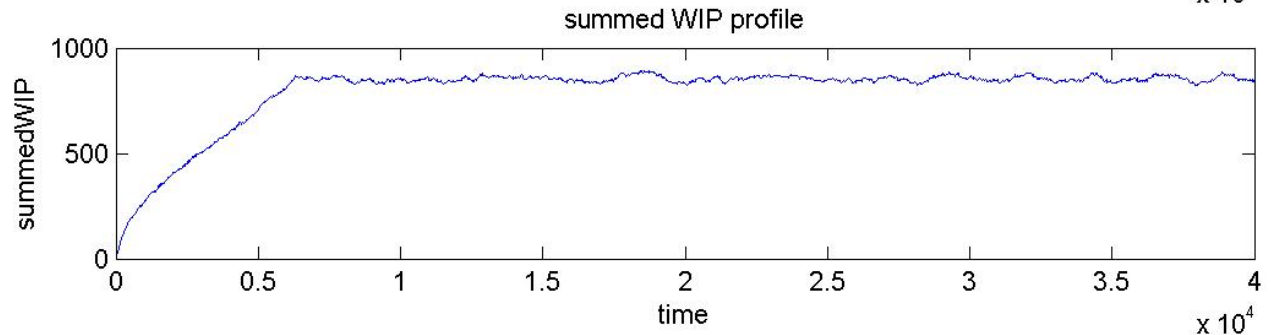
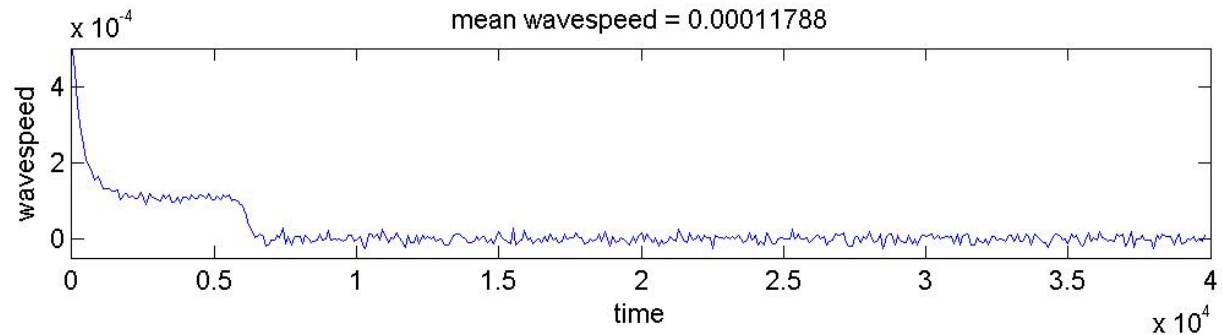
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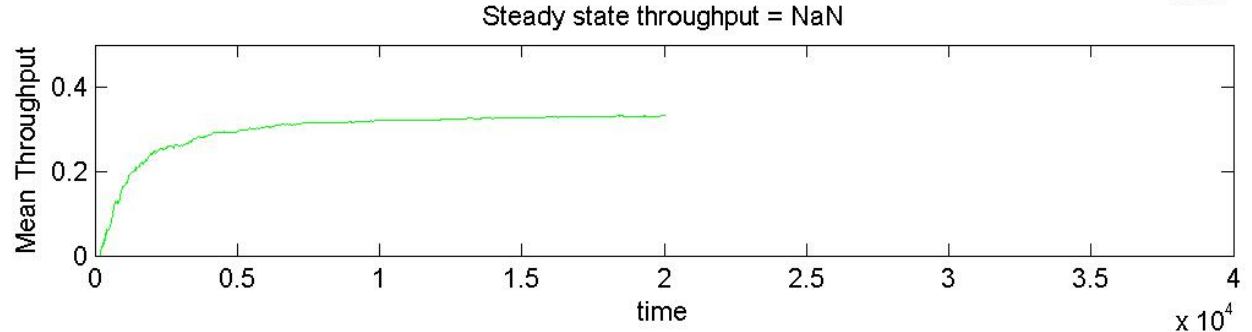
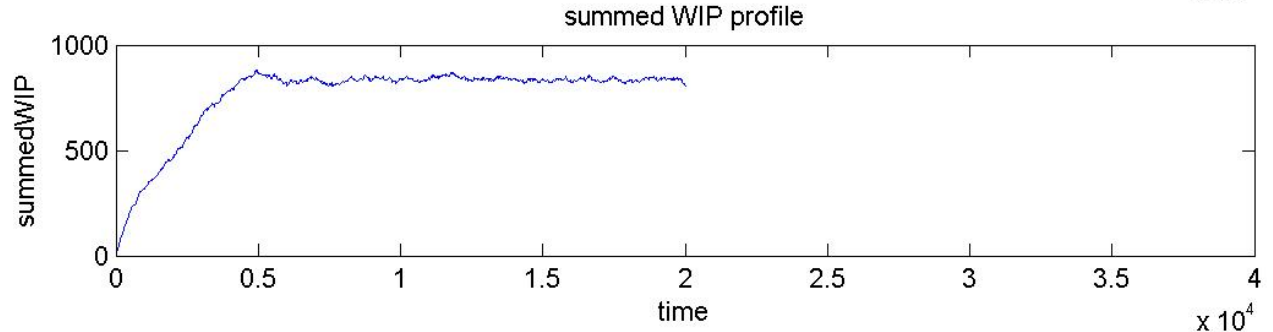
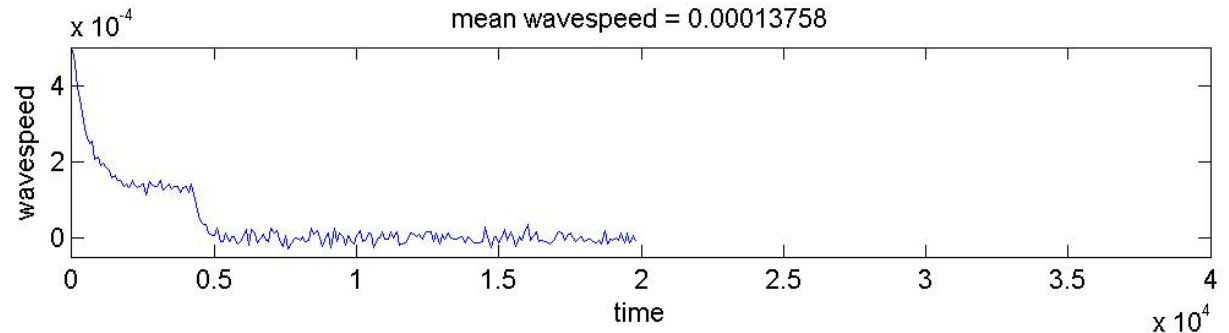
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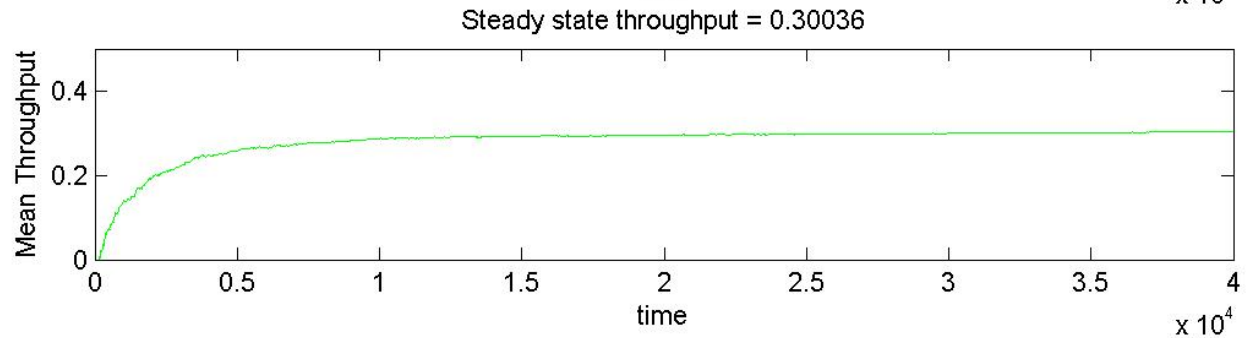
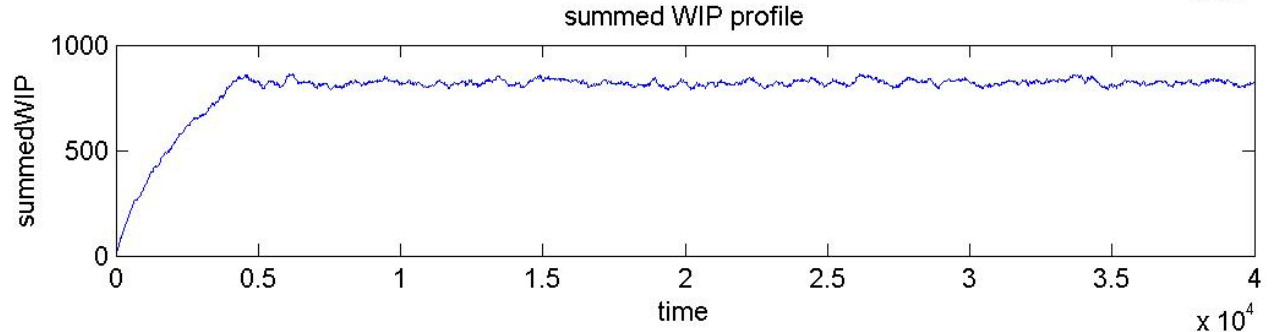
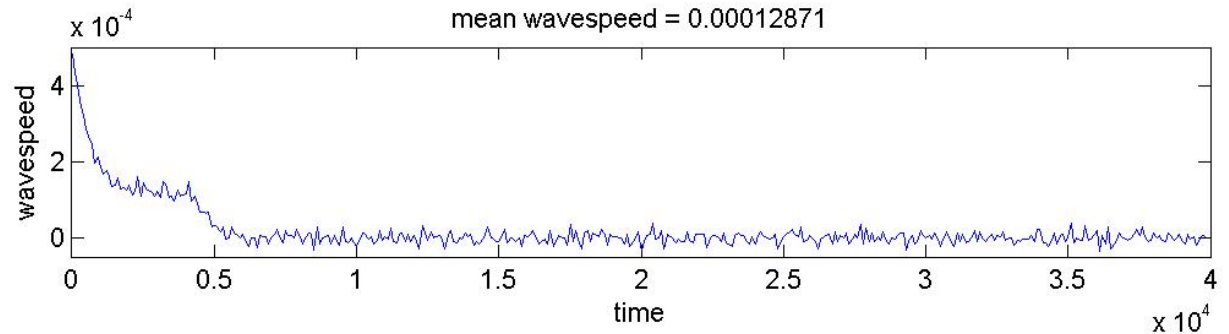
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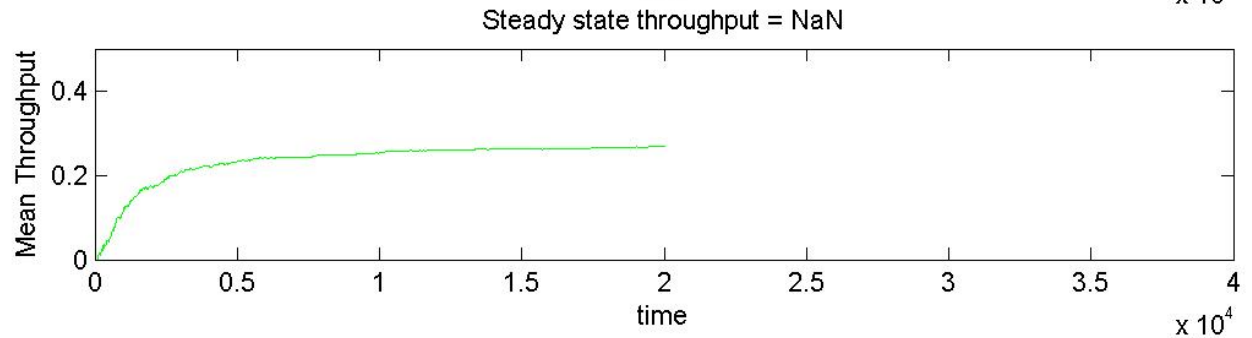
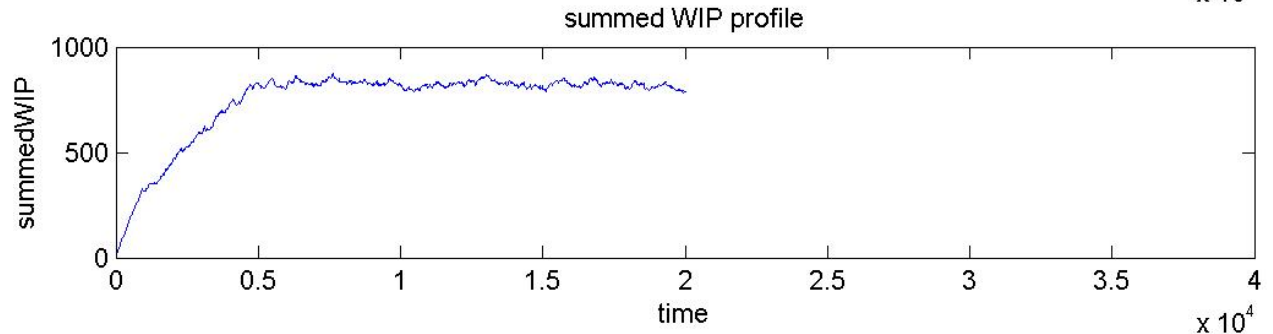
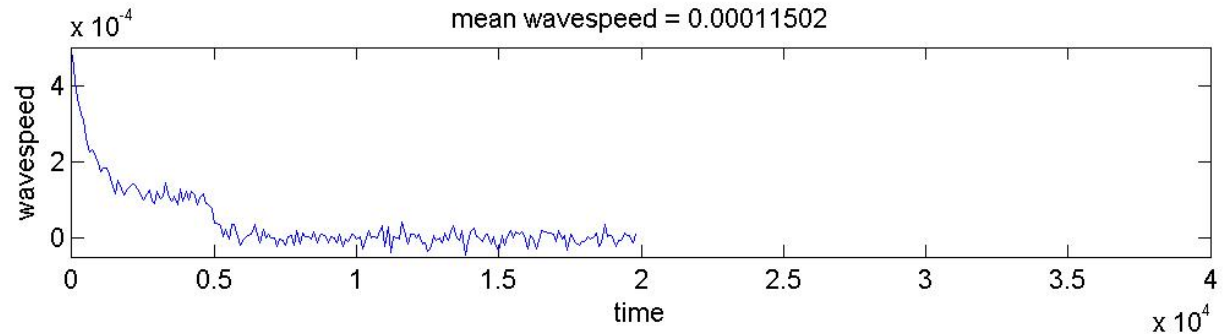
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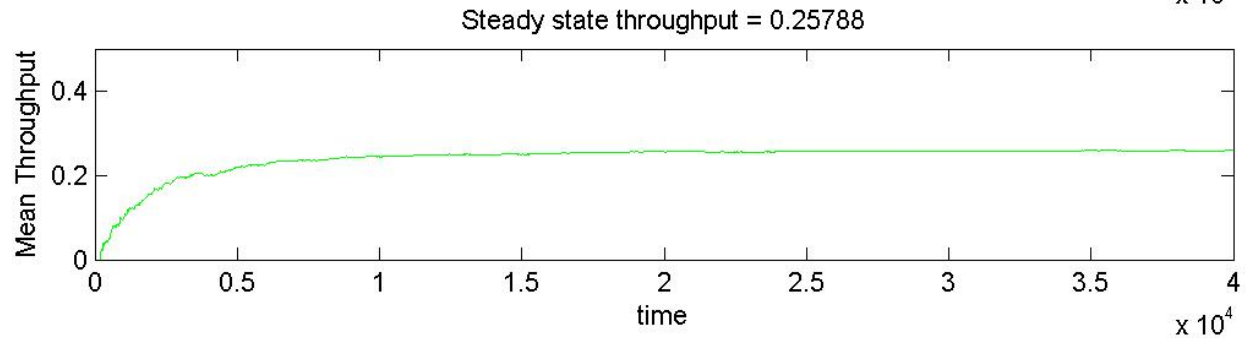
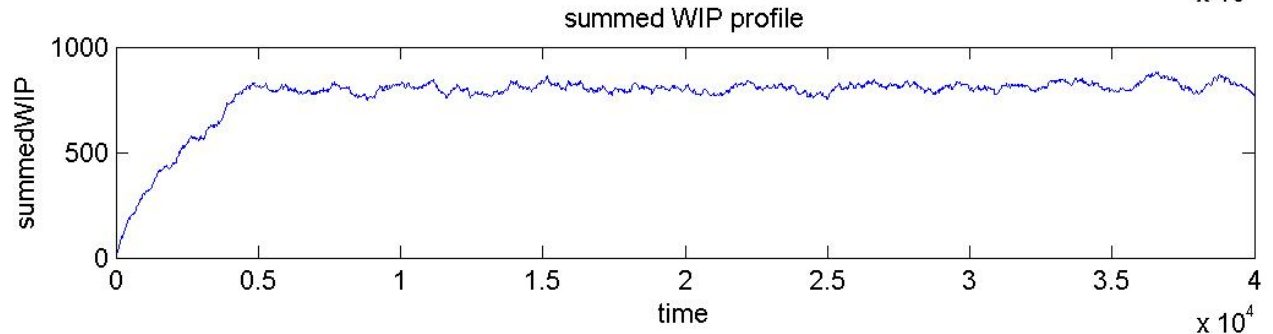
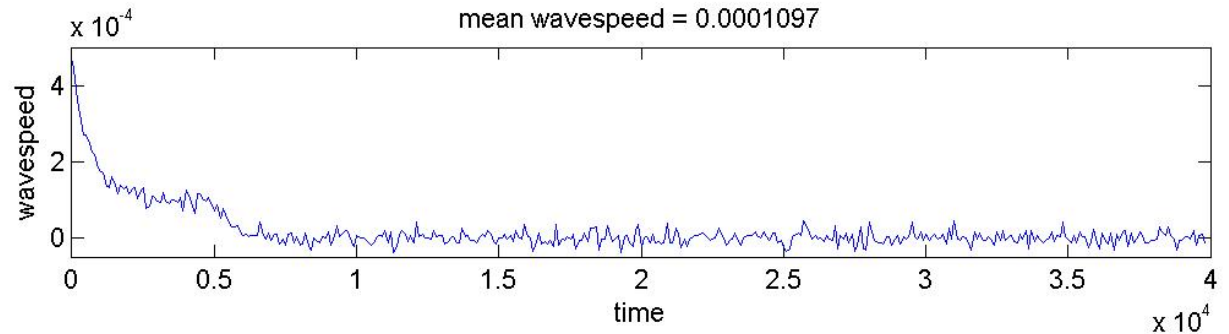
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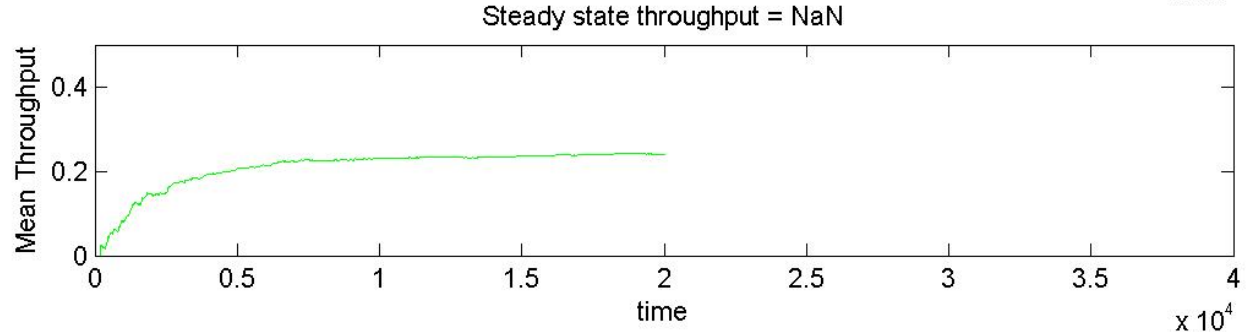
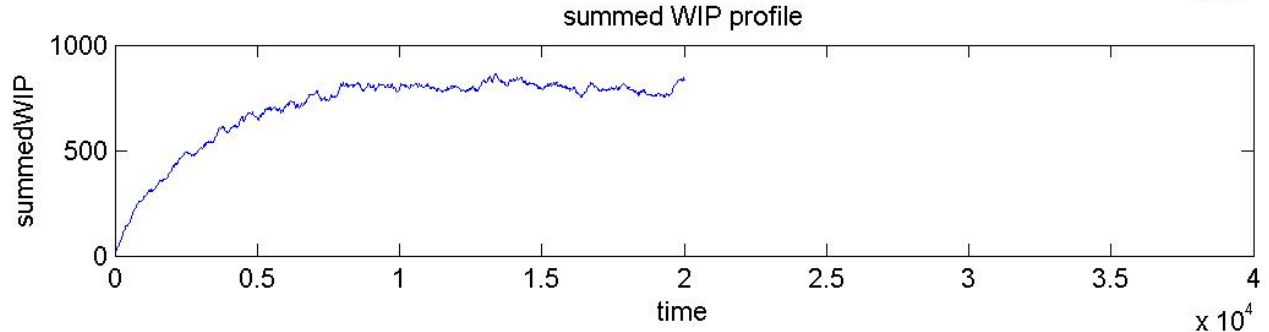
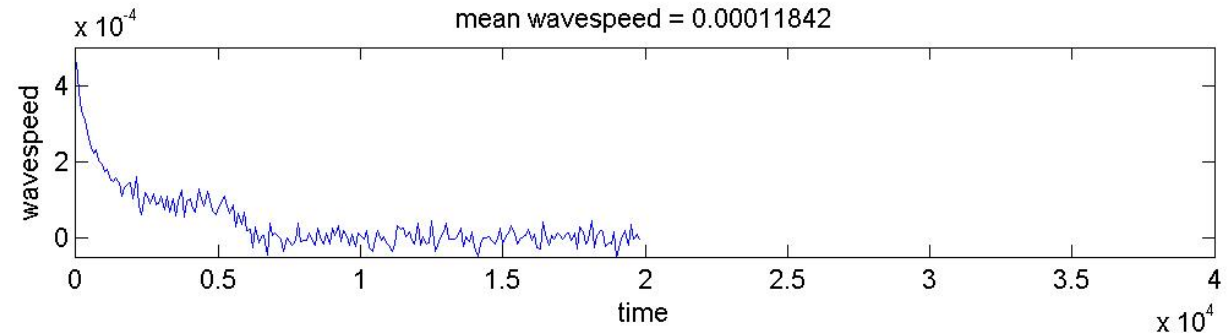
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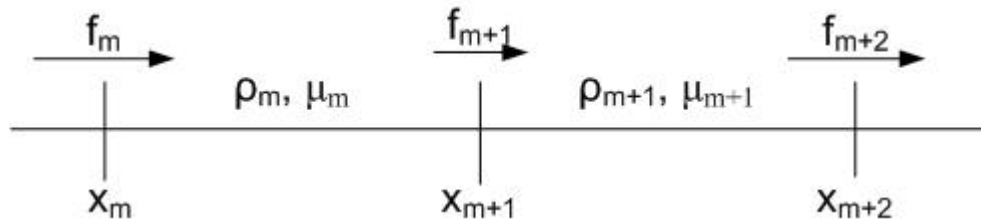


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# PDE

## The first steps from DEM to PDE

## From DEM to PDE: The PDE setup



$m$  : workstation number

$t$  : time

$f_{(m,t)}$  : flux of flow into workstation  $m$  at time  $t$

$f_{(m+1,t)}$  : flux of flow out of workstation  $m$  at time  $t$  (and into workstation  $m+1$ )

$\rho_{(m,t)}$  : density of workstation  $m$  at time  $t$

$\mu_m$  : process rate (or release rate) of workstation  $m$

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# Behavior

- Items arrive at  $x=0$  and proceed to completion (right) as governed by flux conservation laws.
- Movement also determined by clearing functions and immediate downstream and upstream neighbor information.
- CFL conditions are applied

# Discretization

- Take flux conservation law:

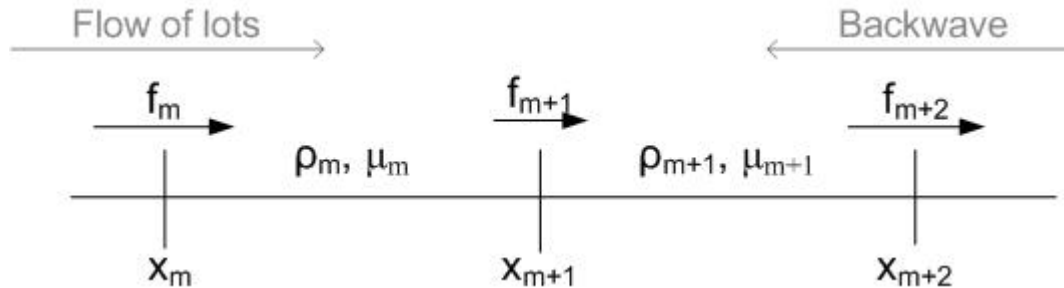
$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial f(x, t)}{\partial x} = 0 \quad (2)$$

- Divide the x-axis into m intervals and discretize:

$$\rho_{(m,t+1)} = \rho_{(m,t)} + \frac{\Delta t}{\Delta m} [f_{(m,t)} - f_{(m+1,t)}] \quad (3)$$

- Now calculate  $f(m, t + 1)$ , for given  $\rho_{(m,t)} \forall m$ .

# Calculation of flux



- Starving:

$$f_{(m+1,t)}^s = \min[\mu_{(m)}, \frac{\Delta m}{\Delta t} \rho_{(m-1,t)}] \quad (4)$$

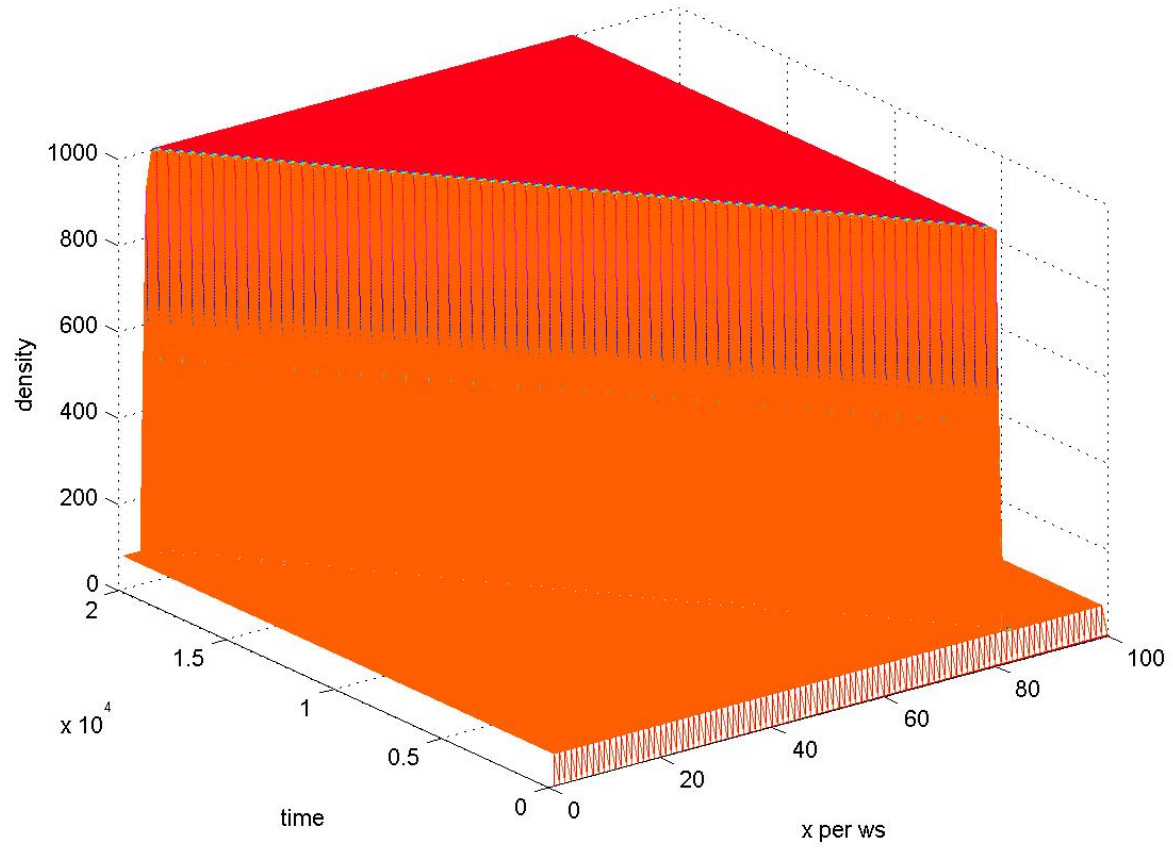
- Blocking:

$$f_{(m+1,t)}^b = \min[f_{(m+1,t)}^s, \frac{\Delta m}{\Delta t} (\rho_{max} - \rho_{(m+1,t)}) + f_{(m+2,t)}^b] \quad (5)$$

- Assume:  $f_{(m+2,t)} = f_{(m+2,t-1)}$

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# Backwave of (discrete) PDE model



# Calculation of flux: Different approach

- Starving:

$$Q_{(e,m)}[\rho_i(t)] = Q_{i,max}[1 - e^{-b_i\rho_i(t)}] \quad (6)$$

$$0 \leq \rho_i(t) \leq \rho_{i,max} \quad (7)$$

- Blocking:

$$f_m(t) = \min[Q_m(t), \frac{\Delta m}{\Delta t}(\rho_{max} - \rho_{m+1}(t)) + f_{m+1}(t)] \quad (8)$$

- Randvoorwaarden:

$$f_M(t) = Q_M(t) \quad (9)$$

$$f_0(t) = \min[\lambda, \frac{\Delta m}{\Delta t}(\rho_{max} - \rho_1(t)) + f_1(t)] \quad (10)$$

$$f_{-1}(t) = \lambda \quad (11)$$

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# Heuristics for deterministic case

$$V_{wave} = \frac{1}{\frac{t_a t_{eH}}{t_{eH} - t_a} (d_{max} - 1) + \frac{t_a - t_{eL}}{t_{eH} - t_a} t_a} \quad (12)$$

# Simulation Results

Consider the following factory parameters:

- inter-arrival time ( $t_a$ ) = 2.0;
- process times: ( $t_e L$ ) = 1.5; ( $t_e H$ ) = 2.1;
- number of machines ( $M$ ) = 100;
- maximum density ( $d_{max}$ ) = 1000 (= 10 lots);
- process time change ( $WSN$ ) = 80;