LECTURE 1: Introduction

PDF available at http://math.la.asu.edu/∼chris/CIME09/CIME09.htm

CONTENTS

- Definition and a generic example.
- Model Hierarchies:
  - Discrete Event Simulation.
  - Rate equations (fluid models).
- Fluxes, Clearing functions and queueing theory.
- Re-entrant systems and simple policies.
- Non - equilibrium theories, delays and conservation laws.

Definition of a supply chain $_{02}$

(Daganzo '02)

One supplier takes an item, processes it, and hands it over to the next supplier.

Suppliers (Items):

- Machines on a factory floor (product item),
- Agent (client),
- Factory, many items,
- Processors in a computing network (information),

A Toy Factory

	diffusion 1	diffusion 2	litho 1	etch clean	etch 1	ion impl	metal dep	litho 2	etch 2	. .	
step	a	b	c	d	e	f	g	h	i		
1				0.25							clean wafer
2	8.00										grow a layer
3			1.00								pattern it
4					1.00						etch away some
5		6.00									grow a layer
6			1.25								pattern it
7						2.50					implant ions
8				0.50							remove mask
9	7.00										grow a layer
10			1.00								pattern it
11					1.00						etch some away
12				0.25							clean wafer
13		5.00									grow a layer
14			1.25								pattern it
15						3.50					implant ions
16				0.50							remove mask

	diffusion 1	diffusion 2	litho 1	etch clean	etch 1	ion impl	metal dep	litho 2	etch 2	. .	
step	a	b	c	d	e	f	g	h	i		
17								1.50			pattern contact
18									1.75		etch contact
19							2.25				layer metal
20								1.00			pattern metal
21									2.25		etch metal
22								1.50			pattern contact
23									2.00		etch contact
24							2.25				layer metal
25								1.00			pattern metal
26									2.50		etch metal
200											lots started per week

AVAILABILITY

diffusion 1	diffusion 2	litho 1	etch clean	etch 1	ion impl	metal dep	litho 2	etch 2		
0.00	0.00	0.00	0.00	0.00	0.00	4.50	5.00	8.50		total hours required per lot
0.00	0.00	0.00	0.00	0.00	0.00	900.00	1000.00	1700.00		total hours needed per week
0.80	0.75	0.90	0.70	0.75	0.85	0.85	0.90	0.65		(average availability)
134.40	126.00	151.20	117.60	126.00	142.80	142.80	151.20	109.20		total hours available per machine per week
0.00	0.00	0.00	0.00	0.00	0.00	6.30	6.61	15.57		tools needed as time req / time avail
1.25	1.25	1.00	2.00	1.50	1.25	1.25	1.10	1.50		degree of constrainedness desired
0.00	0.00	0.00	0.00	0.00	0.00	7.88	7.28	23.35		number of tools needed
0	**0**	**0**	**0**	**0**	**0**	**8**	**8**	**24**		number of tools installed

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Discrete Event Simulation 08

- Multi - agent models.
- Every individual part is modeled separately.
- Stochasticity: requires multiple realizations of the same process and averages.
- Parameterized by events instead of agents.
- Allows for implementation of non - Markovian processes.

Events $E = \{e_1, ..e_K\}$ happening at times $T = \{\tau_1, .., \tau_K\}$
State space $S(t) = \{s_1, .., s_N\}$

1. Find the next event : $\tau_\alpha = \min\{\tau_1, .., \tau_K\}$
2. Carry out the event e_α by changing the state space. $S \longrightarrow F(S)$ (possibly random).
3. Update the schedule. (Update T, e_α might trigger itself and other events.) Choose the new τ's from a distribution.
4. Go to 1 and look for the next event.

- This a big computational problem. $O(10^6)$ variables, $O(10^6)$ events on relevant time scales.
- Needs at least $O(10^2 - 10^3)$ realizations of the underlying stochastic processes.
- Does not render itself easily to optimization.

Rate equations and fluid models $_{13}$

- Each machine modeled by a 'bucket'.
- Content of the 'bucket' (the fluid or the 'WIP', Work in Progress) changes continuously over time.
- Rate equations for the WIP of each station.

$W(t) = (w_1, .., w_M)$: WIP of each station.

$$\boxed{\frac{d}{dt} w_m = \phi_m^{in}(W, \phi^{out}) - \phi_m^{out}(w_m)}$$

$\phi_m^{in}(W)$: input rate, given by $\phi_n^{out}, \ n = 1 : M$ and external inputs of the system.

$\phi_m^{out}(w_m)$: outflux given in terms of w_m.

- ODE system on a network, governed by Kirchhoff's law (mass conservation).
- Question: What is $\phi^{out}(W)$ and why does it only depend on $W(t)$?
- Clearing functions (Graves ('92)): Use steady state queueing theory to model ϕ^{out}.

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Queuing theory $_{16}$

Write ϕ^{out} in terms of w in steady state.

Example: The $M/M/1$ queue.

Markov processes for both arrivals and service.

- λ: arrival rate, $\Delta t \lambda$: probability of arrival in the infinitesimal time interval Δt.
- μ: service rate, probability of service in Δt.
- q: length of the queue.

$$q(t+\Delta t) = \begin{pmatrix} q(t)+1 & P = \Delta t\lambda(1-\Delta t\mu) \\ q(t) & P = (1-\Delta t\lambda)(1-\Delta t\mu) + \Delta t^2\lambda\mu \\ q(t)-1 & P = \Delta t\mu(1-\Delta t\lambda) \end{pmatrix}$$

The discrete probability distribution $_{19}$

$$\boxed{f(w,t) = P[q(t) = w]}$$

$$f(w, t + \Delta t) =$$

$$\Delta t \lambda f(w-1,t) + (1 - \Delta t \lambda - \Delta t \mu) f(w,t) + f(w+1,t) \Delta t \mu + O(\Delta t^2)$$

$\Rightarrow$

$$\boxed{\partial_t f = \lambda f(w-1,t) - (\lambda + \mu) f(w,t) + \mu f(w+1,t)}$$

Rescale for large w: $f(w,t) \to f(hw,t)$

$$\partial_t f = \frac{1}{h^2}[Df(w-h,t)-(2D+\gamma h)f(w,t)+(D+\gamma h)f(w+h,t)]$$

$$\boxed{D = h^2\lambda, \quad \gamma = h(\mu-\lambda)}$$

$$\boxed{\partial_t f = \partial_w[D\partial_w f + \gamma f], \quad D\partial_w f + \gamma f|_{x=0} = 0}$$

Expectations in steady state:

$$f(w) = \frac{\gamma}{D}\exp(-\frac{\gamma}{D}w), \quad \langle w\rangle = \int wf\,dw = \frac{D}{\gamma} = \frac{h\lambda}{\mu-\lambda}$$

Remarks $_{22}$

- Steady state exists only for $\gamma > 0,\ \mu > \lambda$.
- To go to the continuum limit we need $D, \gamma = O(1)$, so

$$\boxed{\mu + \lambda >> \mu - \lambda}$$

  has to hold.

This gives for the flux $\phi^{out} = \phi^{in} = \lambda$

$$\lambda(\langle w \rangle) = \frac{\mu}{1 + \frac{\langle w \rangle}{h}} = \phi^{out}(\frac{\langle w \rangle}{h}) \Rightarrow \phi^{out} = \frac{\mu}{1 + w}$$

- Clearing functions assume a quasi - steady state and full queues.
- There are a multitude of CFs for different sorts of arrival and service processes.

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Topology and Networks $_{24}$

- Model arbitrarily complex re-entrant production networks as flows on a graph.
- Each edge of the graph represents a linear sub - process of the overall production process.

On each arc we have

$$\partial_t w_j = \phi_j^{in} - \phi_j^{out}.$$

Connect influx and outflux by a connectivity matrix (Kirchhoff's law) $\Rightarrow$ yields a system of conservation laws.

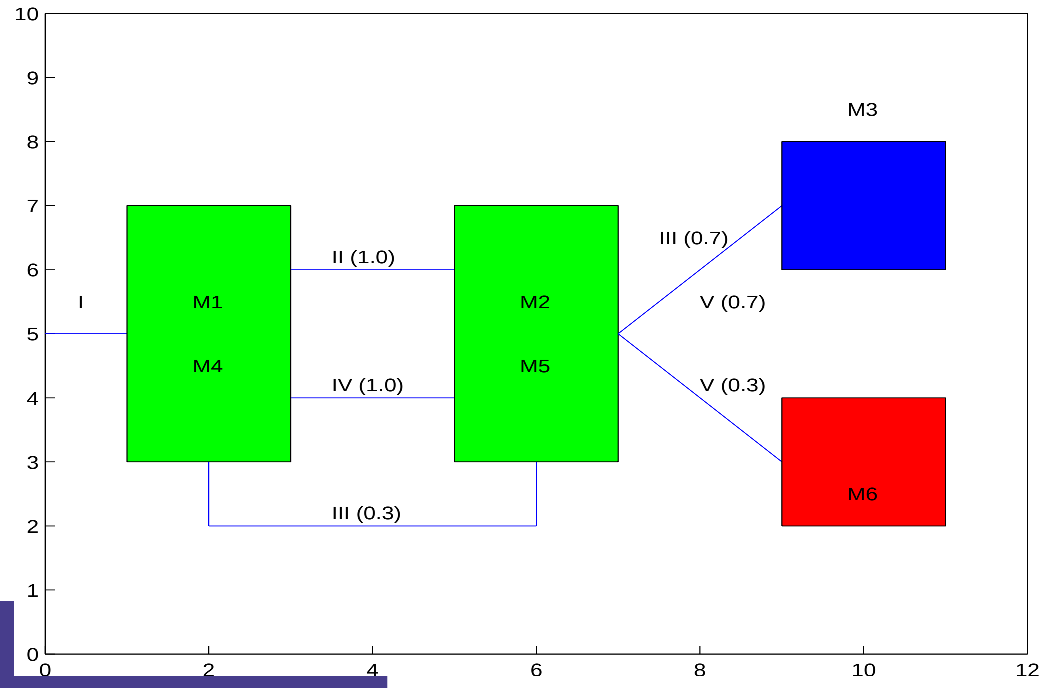
$$\phi_j^{in} = \sum_k A_{jk}(W)\phi_k^{out} + \lambda_j$$

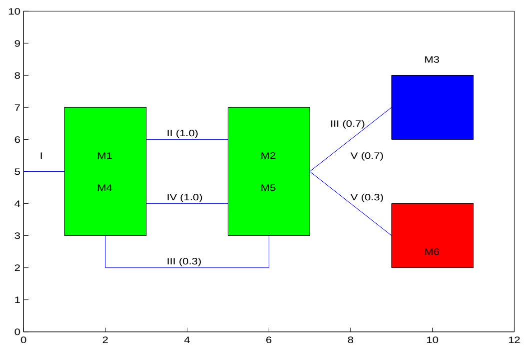
A_{jk}: percentage of the product of sub - process k going into sub - process j.

Re-entrant production lines $_{27}$

- The part has to go through the same machine more than once.
- Model by creating virtual machines .
- This creates the necessity for a policy $A = A(W)$.

Example: Repaint

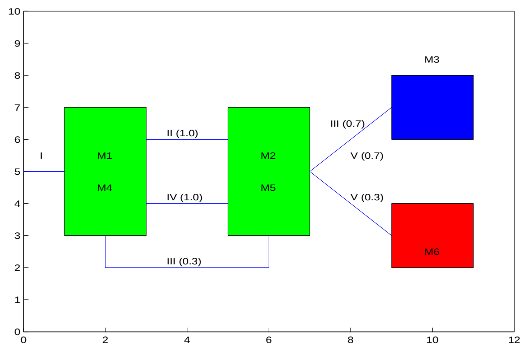




$\longrightarrow$ Regular path: I $\longrightarrow$ II $\longrightarrow$ output bin

$\longrightarrow$ 30 % has to to be done again ,

Identify $I \rightarrow M1, M4, II \rightarrow M2, M5$



$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0 \end{pmatrix}$$

A simple policy: LISFO $_{31}$

Last in System First Out: Parts in the beginning of the process have higher priority.

Give M_1 priority over M_4 and M_2 priority over M_5

$$\mu_1 = c(M_1), \quad \mu_4 = c(M_1) - \phi_1^{out}$$

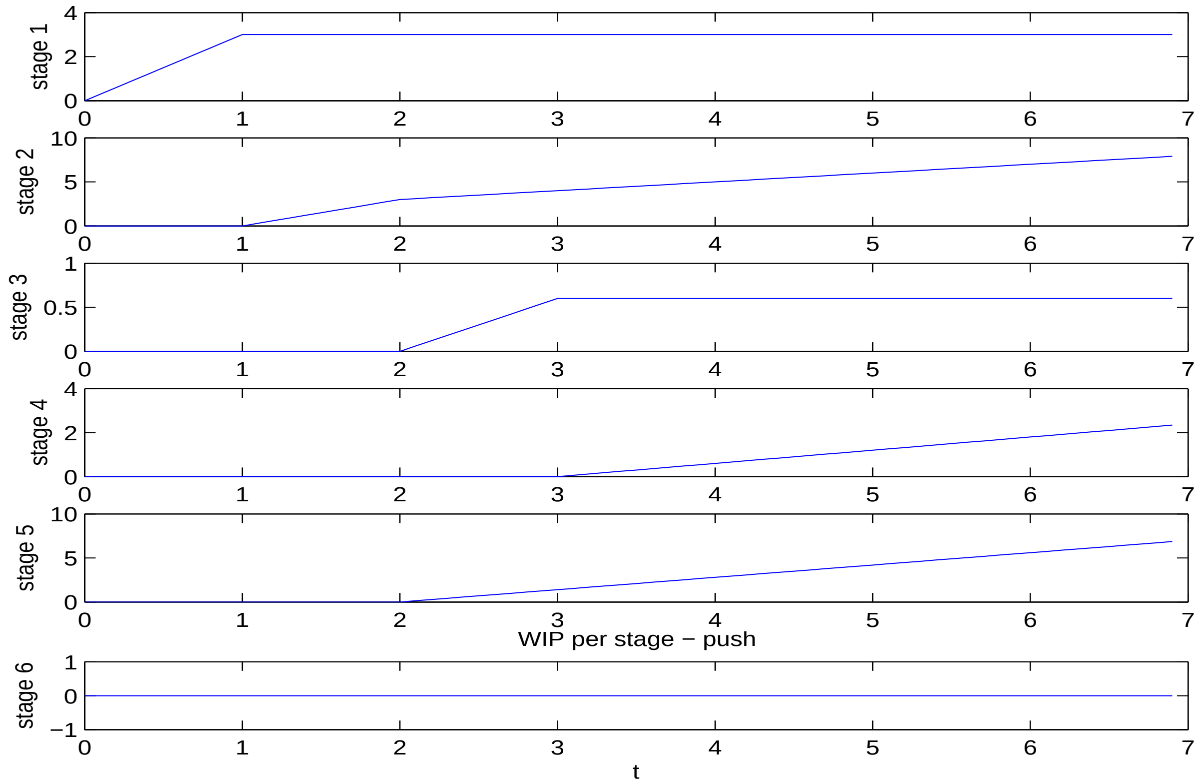
$$\mu_2 = c(M_2), \quad \mu_5 = c(M_2) - \phi_2^{out}$$

The alternative (FISFO):

Parts in the end of the process have higher priority .

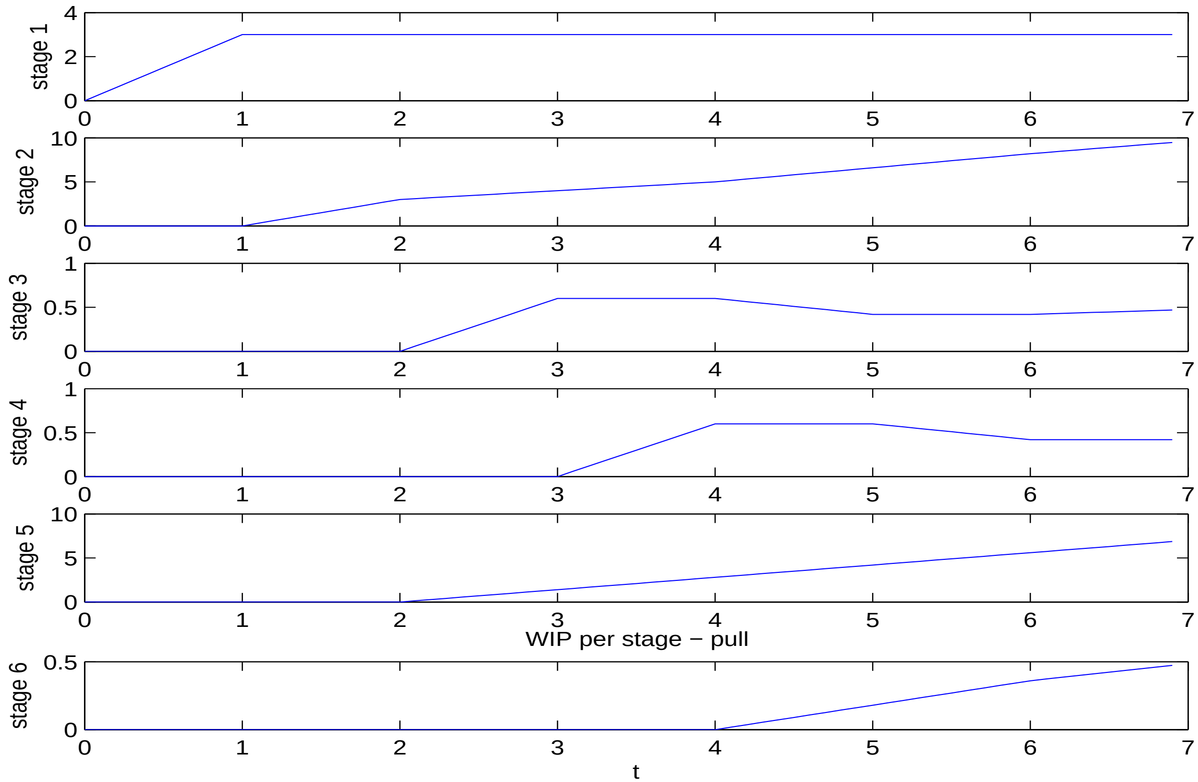
$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0.3 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 \end{pmatrix}$$

LISFO Results - WIP



First pass has higher priority. Influx exceeds temporarily the capacity of $M2 - M5$. No output.

FISFO Results - WIP



Second pass has higher priority. Influx exceeds capacity of M2. Second pass in M2 bounded.

The advantage of clearing functions $_{37}$

$$\boxed{\partial_t w_j = \phi_j^{in} - \phi_j^{out}(W), \quad \phi_j^{in} = \sum_k A_{jk}\phi_k^{in} + \lambda_j}$$

A: connectivity matrix (Kirchhoff's law).

λ_j: external inputs.

- Scalability: Considering more parts results just in higher concentrations.
- Differential equations: Asymptotics and Optimization.

The problem with clearing functions $_{39}$

- They are based on quasi steady state assumptions.
- They do not consider time delays.

$$\frac{d}{dt}w = \phi^{in} - \phi^{out}, \quad \phi^{in}(t) \rightarrow \phi^{in}(t) + \delta(t - t_0)$$

- Gives an immediate response in ϕ^{out}!
- ϕ^{out} computed from the WIP w at the time the part leaves the queue.
- The model should be given by a distributed delay differential equation using the history of $w(t)$ between entrance and exit of the part.

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Conservation Laws $_{41}$

A simple Lagrangian picture:

- Assume the TPT $\tau(s)$ of part number s can be estimated at the entry time $a(s)$.
- Evolve the part on an artificial road $x \in [0, L]$ along the trajectory

$$\partial_t \xi(t,s) = v(s) = \frac{L}{\tau(s)}, \quad \xi(a(s), s) = 0\,.$$

The Eulerian picture:

$$\rho(x,t) = \int \delta(x - \xi(t,s))\, ds$$

Continuity equation:

$$\boxed{\partial_t \rho(x,t) + \partial_x \phi(x,t) = 0, \quad \phi(x,t) = u(x,t)\rho(x,t)}$$

Burger equation:

$$\boxed{\partial_t u + u \partial_x u = 0}$$

as long as parts do not overtake (until the development of shocks).

Boundary conditions:

$$\lambda(a(s)) = \frac{1}{a'(s)} \Rightarrow \phi^{in}(t) = u\rho(x=0,t) = \frac{d}{dt}a^{-1}$$

$$u(x=0,a(s)) = v(s) = \frac{L}{\tau(s)}$$

Proposition $_{45}$

For simple deterministic processing rates there are no shocks in $[0, L]$:

$$\tau(s) = \frac{w(a(s))}{\mu} + T, \quad w(t) = \int_0^L \rho(x,t)\,dx$$

μ: processing rate, T: processing time

Compute the earliest time two parts can meet:

$$w(a(s+\Delta s)) \geq w(a(s)) - \mu[a(s+\Delta s) - a(s)]$$

$$\Rightarrow \tau(s+\Delta s) \geq \tau(s) - a(s+\Delta s) + a(s)$$

$$\xi(t,s) = \frac{L}{\tau(s)}(t - a(s))$$

$$\boxed{\xi(t,s+\Delta s) = \xi(t,s) \Rightarrow \xi(t,s) \geq L}$$

Summary:

- An abrupt change in the influx ϕ^{in} results in a time delayed change in the outflux.
- PDE conservation laws are a way to encode the (distributed) delay.
- Leads to conservation laws on graphs for production networks.
- More complicated conservation law models could be derived using more complicated queueing models.
- This still does not take into account fluctuations in the processing rate μ.