Matrix Operations on a Casio Graphing Calculator

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The use of a graphing calculator can be useful and convenient, especially when reducing a matrix that has entries with many decimal places. The inverse of a matrix can also be found easily. One of the homework assignments for MAT 119 is to reduce a matrix with a graphing calculator. Chris Heckman will demonstrate how to perform row operations with a Casio calculator.

Introduction

Two important applications with matrices in MAT 119 are solving a system of linear equations and finding the inverse of a matrix. The Casio series of graphing calculators is able to find the inverse automatically, but putting a matrix in reduced row echelon form has to be done manually. The purpose of this paper is to indicate the appropriate steps.

A brief word on notation: When a box is drawn around a symbol, that means to press that key on the Casio. Hence [×] means to push the multiplcation button. If two key presses are used, they will be to refer to functions printed in different colored keys on the calcuator. The keypresses will be given, along with the operation to be performed, the latter in *slanted type* and in brackets. For instance, [SHIFT] [ln] [e^x] means to press [SHIFT], then [ln], and this accesses the function e^x; this is in yellow ink on the Casio 9850GB+. Also, [ALPHA] [X,θ,T] [*A*] puts the calculator in alphabetic mode and enters the letter A in the display; these letters are printed in red ink on the Casio 9850GB+.

Colors may vary from model to model; other versions may have a [2nd] key instead of the [SHIFT] key; and functions may be located on other keys. Consult your manual or calculator for further details.

Solving A System of Linear Equations

Suppose you want to solve the system:

$$\begin{array}{rcrcrcr} 25x & + & 61y & - & 12z & = & 10 \\ 18x & - & 12y & + & 7z & = & -9 \\ 3x & + & 4y & - & z & = & 12 \end{array}$$

Here's how to solve it.

1. *Enter the augmented matrix into the calculator.*

Press [MENU] to bring up the main menu, then either use the arrow keys to move the cursor (the darkened square) to MAT mode, or press the [3] button.

You now see a list of matrices. A darkened line is once again your cursor; you can move it up or down by pressing the arrow keys. When you find a place you want to put your augmented matrix into, enter the dimensions of the matrix you want. In our case, press [3] [EXE] [4] [EXE] to create a 3×4 matrix. (The Casio accepts dimensions of up to 100.)

NOTE. *At any time, if you realize you made a mistake, you can generally go back a step by pressing the* [EXIT] *key.*

Now you need to enter the coefficients and numbers on the right hand side. The cursor is placed in the first row, first column of the matrix. If you type in a number (or an expression) and hit [EXE], it will put that number into the matrix and move to the next position, to the right (if you're not at the right-most column) or down to the first column of the next row. After entering a number in the last row and last column, the cursor stays there.

So we need to put in the coefficients. The keystrokes for this are [2] [5] [EXE] [6] [1] [EXE] [-] [1] [2] [EXE] [1] [0] [EXE], etc.[1]

2. *Perform the row operations.*

If you look at the calculator's display, you will see three boxes at the bottom, labelled R-OP, ROW, and COL. ROW and COL are for manipulating the matrix one row or column at a time, and the R-OP is used for performing row operations. Press the key right below it on the calculator, [F1].

You now get four other boxes, SWAP, XRW, XRW+, and RW+. These boxes are for swapping (interchanging) two rows, multiplying a row by a number, adding a multiple of one row to another, and adding a row to another, respectively.[2] Pressing the keys below them ([F1], [F2], [F3], or [F4]) will bring up another screen, asking for particular values, and then the row operation will be performed.

We will start by swapping the first and third rows.[3] Press [F1]. The calculator brings up a a line at the bottom of the screen:

$$\texttt{Swap Row m} \leftrightarrow \texttt{Row n},$$

and right above it is `m?`. The calculator is asking for the value of m, and is showing you where it will be used. Since we want to swap the first and third rows, press [1] [EXE] — now the calculator will ask for n — [3] [EXE].[4]

This has swapped the two rows. Now we want to divide the first row by 3, so that there is a 1 in the upper left corner. This is what the XRW operation is for; press [F2] now. You will now see

$$\texttt{k} \times \texttt{Row m} \rightarrow \texttt{Row m}$$

Since we want to divide by 3, $k = \frac{1}{3}$, but if we enter [1] [÷] [3], the Casio will round off this number to 0.33333, which is bad, since row reduction is extremely sensative to round-off error. Instead, we will use the Casio's capability of saving fractions exactly. Enter [1] [$a\ b/c$] [3] [EXE] instead. Then enter [1] [EXE] to select row 1.

The entries of the first row change to 1, 1.3333, −0.333, and 4. It looks like we have to deal with round-off error anyway, but if we move the cursor up to the first row and third column (press [↑] [↑] [←]), we see `-1,3` in the lower right corner. This means that the entry in the first row, third column, is really $\frac{-1}{3}$, and is being stored that way. The approximation is only used because the matrix only has five "characters" for each entry.[5]

If you press [←] again, you see that that entry is `1,1,3`, which is $1\,\frac{1}{3}$. To get the exact value as an improper fraction, press [SHIFT] [$a\ b/c$] [d/c].

Now you need to subtract 18 times Row 1 from Row 2, to turn the entry in the 2nd row, 1st column into a 0. Press: [F3], and you now see

$$\texttt{k} \times \texttt{Row m} + \texttt{Row n} \rightarrow \texttt{Row n}$$

You need to enter k: [-] [1] [8] [EXE], m: [1] [EXE], n: [2] [EXE]. Note that the entries in the second, third, and fourth columns are exactly −36, 13, and −81.

Now, you perform these same types of row operations over and over. The key sequences are below.

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1 And so on for the other rows. Note that you use the unary minus to enter negative numbers.

2 Think of RW+ as adding 1 times a row to another.

3 You don't have to do this to solve the system; it is just for a demonstration of how to use the calculator.

4 Or [3] [EXE] [1] [EXE], if you prefer.

5 If you have a matrix with a large number of rows or columns, you can keep pressing the arrow keys to bring the other areas into view.

1. `F3` `-` `2` `5` `EXE` `1` `EXE` `3` `EXE`
2. `F2` `-` `1` $a\ {}^b\!/_c$ `3` `6` `EXE` `2` `EXE`
3. `F3` `-` `4` $a\ {}^b\!/_c$ `3` `EXE` `2` `EXE` `1` `EXE`
4. `F3` `-` `8` `3` $a\ {}^b\!/_c$ `3` `EXE`[6] `2` `EXE` `3` `EXE`
5. `F2` `1` `0` `8` $a\ {}^b\!/_c$ `6` `8` `3` `EXE` `3` `EXE`
6. `F3` `1` `3` $a\ {}^b\!/_c$ `3` `6` `EXE` `3` `EXE` `2` `EXE`
7. `F3` `-` `4` $a\ {}^b\!/_c$ `2` `7` `EXE` `3` `EXE` `1` `EXE`

Now the matrix is in reduced row echelon form. Move the cursor to the fourth column, first row to find the value of $x = \frac{3119}{683}$, down to the second row to find $y = -\frac{4401}{683}$, and down again to find $z = -\frac{16{,}443}{683}$.

Inverting a Matrix

Inverting a matrix can be done on the Casio without as much work; it is built-in to the calculator. Here we will invert the matrix $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$.

1. *Enter the matrix into the calculator.*

This can be done as in the previous section, or you can go to the RUN MODE (`MENU` `1`) and enter the matrix there. If you are using the MAT mode, you need to choose a new matrix. Otherwise, type: `SHIFT` `+` [*[*] `SHIFT` `+` [*[*] `1` `,` `1` `,` `-` `1` `SHIFT` `−`[7] [*]*] `SHIFT` `+` [*[*] `2` `,` `1` `,` `1` `SHIFT` `−` [*]*] `SHIFT` `+` [*[*] `1` `,` `0` `,` `1` `SHIFT` `−` [*]*] `SHIFT` `−` [*]*]. This is the matrix. Now to store it, type the assignment key `⟶`[8] and then `OPTN` `F2` `F1` [*Mat*],[9] then `ALPHA` `log` [*B*]. You should have the following in your display:

[[1,1,-1][2,1,1][1,0,1]] ⟶ Mat B

Now hit `EXE`. The Casio shows you the matrix you just entered.

2. *Calculate the inverse.*

Change to run mode (`MENU` `1`) if you entered the matrix the same way you did it for solving a system. Now you can calculate the inverse by pressing `OPTN` `F2` `F1` [*Mat*] `ALPHA` `log` [*B*] `SHIFT` `)` [$^{-1}$] `EXE`, namely `Mat B`$^{-1}$. The answer is $\begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{pmatrix}$, after a few seconds have gone by.

Other matrix computations are possible. For instance, to find B^{10}, type `OPTN` `F2` `F1` [*Mat*] `ALPHA` `log` [*B*] `^` `1` `0` `EXE`; the answer is $\begin{pmatrix} 1897 & 1432 & -351 \\ 3945 & 2978 & -730 \\ 1432 & 1081 & -265 \end{pmatrix}$.

[6] Using the arrow keys, move the cursor to the third row, second column. The exact value of this entry is `-83,3`.

[7] This is the binary minus key, used for subtracting one number from another.

[8] which is not the arrow key used to move the cursor around

[9] This is to indicate that the object which follows should be treated like a matrix, in case you want to do matrix arithmetic. Otherwise scalar multiplication could not be distinguished from matrix multiplication.